Agent's minimal intelligence calibration for realistic market dynamics

Iryna VERYZHENKO and Olivier BRANDOUY and Philippe MATHIEU

Abstract This paper investigates the question of the sophistication level, in the mean of behavior and intelligence, one should endow artificial traders in order to obtain, with realistic market microstructure, not only both qualitative and quantitative stylized facts in an artificial market. For this purpose, we introduce an agent-based simulation environment with an architecture close to the Euronext-NYSE Stock Exchange. Series of experiments with different kinds of agents' behavior and trading framework specifications where realized within this environment. The results indicate that only special calibrations provide realistic stylized facts with coherent quantitative levels. We introduce a new type of agents, called in this paper "strongly calibrated agents", with their specific environment design, that provide price dynamics in quantitative and qualitative accordance with real stock market characteristics.

1 Introduction

Agent-Based Finance, and specifically, Agent-Based Artificial Stock Markets (hereafter ABASM) is an ever-growing field that appears, in the aftermath of the recent financial crisis, as a potential source for renewed analysis concerning the stability of the whole financial system. For example, policy experiments with agent based platforms become more realistic with the increasing sophistication of these softwares, and topics like the assessment of Tobin Tax regarding financial markets liquidity and volatility [9] or the analysis of the linkage market-microstructure and price dynamics [11] can actually be undertaken. One strong argument pleading for an increasing

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role of ABASM in the academics or policy-makers toolbox, is that these softwares can duplicate the main stylized facts that can be observed on real-world stock exchanges (see for a detailed review of stylized facts [5], and for a work involving ABASM in their emergence [6]). In this perspective, several research articles have argued that Zero Intelligence Traders (ZIT, ”à la” [7]) are sufficient to produce such stylized facts (for instance [15]). Nevertheless, these stylized facts remain mainly *qualitatively* congruent with real-world observations and the underlying price motions emerging from ZIT interaction remain, to our opinion, rather unrealistic. If one wants to go beyond this mere qualitative approach, ABASM need some ”calibration” to deliver price dynamics that *quantitatively* correspond to real financial markets motions on the one hand, and to produce more realistic price trajectories on the other hand. Such ”calibration” must be done at the agents level and matched against real-world data. Furthermore, this process must also be grounded on a realistic market microstructure. Thus, the following question is at the bottom line of the present article : *”How basic artificial traders could be for realistic market simulations ?”*.

We show, using an asynchronous Agent-Based platform benchmarked against a major European stock market, that pure ZIT cannot reproduce realistic price dynamics especially when one focus on their quantitative values. We introduce an augmented intelligence specification that aim at delivering both qualitative and quantitative stylized facts and discuss previous results that supported this minimalist specification in Agent's artificial intelligence.

The article is organized as follows. In a first section, we briefly review the relevant literature and point out the main results linking agents intelligence and financial price dynamics. We then describe the agent-based platform used for the experiments run in section 3, and the procedure that was used to verify the ability at mimicking its benchmark real stock market. In a fourth section, we describe our empirical strategy and present the main results we obtain.

2 Literature Review

2.1 Seminal contribution and initial controversy

The minimal agents intelligence calibration became controversial when the much cited Gode and Sunder [7] model faced strong criticisms from Cliff and Bruten [4]. The main concern of this latter research was to evaluate how much intelligence is actually needed in agent to achieve high-level trading performance. Cliff and Bruten calibrated their system through agent's ability to adjust their profits in order to achieve a ”realistic market efficiency”. Other kinds of calibrations have appeared afterwards. For example, some researchers tried to mix agents populations in their models to get stylized facts from behavior heterogeneity (see for example [1]). Later Maslov [10] improved an existing model using limit and market orders. This method has also been employed in the work of [12]. But their models reproduce only some of generic features: namely, a congruent Hurst exponent and fat tails in the return distribution. Challet and Stinchcombe [3] show that in continuous-double auction setting the model of three processes (orders, executions, cancellations) is required to produce the fat-tails and volatility clustering.

In this research we also focus attention on price dynamics itself : this point is usually ignored by authors although we believe it is an important validation instrument for market simulator success : amazingly, one can obtain stylized facts that match "at a qualitative level", real world dynamics with an underlying price series that is totally unrealistic at a first glance. Thus, even if one can observe stylized facts from the return series delivered by the simulator, he/she can easily face a problem of (unrealistic) highly volatile price series for example. Therefore, computer simulations that provide realistic stable price dynamics are particularly interesting to our opinion. The Minimal Market Model (MMM) [2] is, with respect to the latter criterion, particularly promising. These authors claim that this simple model can reproduce real market features in both price and return terms. Voit [13] also programed this model and tried to calibrate it. Simulations end up with very volatile price series. All simplifications failed at stabilizing this highly sensitive system.

One can notice that many research claim that stylized facts, like volatility clustering, positive correlation in order types or shape of the order book, are not directly driven by strategic behavior : the necessary and sufficient ingredients to generate these statistical properties could be a specific market microstructure and zero-intelligence agents. For instance, an exhaustive investigation has been done by Li-Calzi, Pellizzari and Dal Forno ([8], [11]), who show that the choice of a protocol may have a substantial impact on the allocative effectiveness, and other criteria such as excess volume or price dispersion. Nevertheless, and to the best of our knowledge, no paper focuses on the calibration of ABASM such to obtain quantitative stylized facts and non-volatile price dynamic in line with real markets. Our target is to fill this gap and to propose simple parametric methods to calibrate agents behavior to provide realistic price and return dynamics in the ArTificial Open Market API (here-after ATOM, see `http://atom.univ-lille1.fr`). Moving from non-strategic behavior to simple intelligence elements we show that *any assumption about any kind of intelligence has an impact to stylized facts*. We first present the ATOM API, then introduce agents behavior specification within this environment.

3 ATOM and Real World Market

ATOM is a general environment for Agent-Based simulations of stock markets. It is based on an architecture close to the Euronext-NYSE Stock Exchange one. Agent-Based artificial stock markets aim at matching orders sent by virtual traders to fix quotation prices. Price formation is ruled by a negotiation system between sellers and buyers based on an asynchronous, double auction mechanism structured in an order book. Using this API, one can generate, play or replay order flows (what-ever the origin of these order flows, real world or virtual agents population). One of the main advantages of ATOM consists in its modularity. This means that it can be viewed as a system where three interacting main components interact: i) *Agents*, and their behaviors, ii) *Markets* defined in terms of microstructure and iii) the *Artificial Economic World* (including an information engine and, potentially, several economic institutions such as banks, brokers, dealers...). The two first components can be used independently or together. Depending upon the researcher targets, the *Artificial Economic World* can be plugged or not in the simulations. For example, one can use the system for the evaluation of new regulation policies or market procedures, for assessing potential effects of taxes or new trading strategies in a sophisticated artificial financial environment. Thanks to its high modularity and its ability to mimic real-world environments, it can also serve as a research tool in Portfolio Management, Algorithmic Trading or Risk Management among others. From a pure technological point of view, ATOM can also be viewed as an order-flow replay engine. This means that bankers can test their algorithmic-trading strategies using historical data without modifying the existing price series or backtest the impact of their trading-agents in totally new price motions or market regimes generated by artificial traders. Several distinctive aspects of ATOM can be highlighted:

1. It can be used without any agent. One can directly send orders written in a text file (for example, a set of orders as it arrived on a given day, for a given real-world stock market) to each order-book implemented in the simulation. In this case, ATOM serves as a ”replay-engine” and simulations merely rely on market microstructure. It therefore runs really fast (an entire day of trading in less than 5 seconds).
2. ATOM can use various kind of sophisticated agents with their own behaviors and intelligence. Thousands of these agents can evolve simultaneously, creating a truly heterogeneous population. Once designed, agents evolve by themselves, learning and adapting to their (financial) environment.
3. ATOM can mix human-beings and artificial traders in a single market using its network capabilities. This allows for a wide variety of configurations, from ”experimental finance” classrooms with students, to competing strategies run independently. The scheduler can be set so to allow human agents to freeze the market during their decision process or not.
4. ATOM has been tested rigorously. It has the ability to replay perfectly an order flow actually sent to a given market with the same microstructure. The resulting price series (on the one hand, the ”real-world” one and on the other hand, the ”artificial” one) overlap perfectly. Moreover, given a population of agents, ATOM can generate stylized facts qualitatively similar to the market it is geared at mimicking.

Simulations in ATOM are organized as ”round table discussions” and are based on an *equitably random* scheduler. Within every ”round table discussion”, agents are randomly interrogated using a uniformly distributed order. This latter feature ensures that each of them has an equal *possibility* of expressing its intentions. Notice that the API offers a random generator that is shared by all agents. The reproducibility of experiments is therefore guaranteed.

In real life, investors do not share the same attitudes. Some will be more reactive than others, or will implement more complex strategies leading to a higher rate of activity. In ATOM having the possibility to express an intention does not necessarily imply that a new order is issued. Since agents are autonomous, they always have the possibility to decline this opportunity.

Moreover, if an agent had been allowed to send several orders when interrogated, this would have led to an equality problem similar to the one described above. To overcome this issue, agents are just allowed to send at most *one single* order to *a given order book* (*i.e.* one order at most per stock) within the same "round table discussion". However, if an agent plans to issue several orders concerning the same stock (thus, the same order book), she must act as a finite state automata.

ATOM can include human-beings in the simulation loop. A human agent is an interface allowing for human-machine interaction. Through this interface one can create and send orders. Notice that human agents do not have any artificial intelligence : they just embed human intelligence in a formalism that is accepted by the system. To allow the introduction of human in the loop, ATOM has been designed to deal with communications over the network.

4 Empirical strategy and results

4.1 Data description

Our data consist in intraday prices observed in the Euronext Paris Stock Exchange. The sampling of these observations is based on intervals of about 1 minute. We use 37 stocks for January 1st 2001 - January 31th 2001 and August 1st 2002 - August 31th 2002 (in total 1628 assets' price lists). Each day has from 1000 to 5000 records for different assets, depending on the market activity. Our goal is to compare, using price and return series, the results delivered by the ATOM platform under some set of Artificial Intelligence specifications, and real data. For this reason, we first present some general elements, then how agents behavior is progressively modified moving in the direction of growing intelligence.

4.2 Calibration elements: agent's behavior

This section illustrates how we create different agents behavior and how to specify a general environment within the ATOM framework. We first start from a simple model (inspired by [10]), then step by step additional constraints are introduced in order to observe the appearance of nontrivial stylized facts and more realistic price dynamics. Some initial settings, that are implemented in the *Market* component of ATOM, are detailed below.

· Buy and Sell orders arouse with equal probability.
· Each agent can submit both orders, Buy and Sell.
· There are three possible order types: limit, market and cancel. We use also a proportion between different order types, such as 80% of limit orders, 15% of market orders and 5% of cancel orders. These proportions are equal to those observed for one specific asset within one specific day. This initial calibration is geared at imposing realistic market conditions, where both market and limit orders come in different sizes and exist for various time frame.
· Transactions are realized for a single asset.
· There are two types of traders regarding the volume that they are able to set up in the orders. A first subset, in which *"Big fish"* traders send orders with a volume close to the maximum possible value (initialization parameter), and *"Small fish"* agents, respectively with a volume close to the minimum.
· Budget constraints are implemented: traders cannot make a trade that will yield a negative profit, *i.e.*, buyers cannot buy at a price higher than their buyer value (reservation price) and sellers cannot sell for a price below their seller cost.
· Parameters for setting initialization are calculated based on the real market data. These parameters are measured for each stock within each day.

We now introduce a detailed description of agents behavior, moving from uncalibrated to strongly calibrated agents, in other words the agents in growing intelligence. These agents are realized in the *Agents* component of the ATOM API.

- *Uncalibrated agents* should be considered in the above predefined framework design. They pick a log-normally distributed price $\alpha(t) \sim Log-N(P_{mean}, P_{sd})$, where P_{mean} and P_{sd} are respectively mean value and standard deviation of real market data (initialization parameters). Volume is an integer drawn normally within the predefined (as parameters) ranges.
- *Statistically calibrated agents* have ranges within which they are able to setup orders' price and volume. Let $[P_{min}, P_{max}]$ be respectively the minimum and maximum real market intraday prices, and $[V_{min}, V_{max}]$ the minimum and maximum experienced trading volume for one specific asset. If the agent is willing to send an Ask order, he/she should respect a sellers' range $[P_{A_{min}}, P_{A_{max}}]$; respectively, in case of a Bid order, the price will be within the limits $[P_{B_{min}}, P_{B_{max}}]$, where $P_{min} \leqq P_{B_{min}} \leqq P_{A_{min}} \leqq P_{B_{max}} \leqq P_{A_{max}} \leqq P_{max}$. The price is described in the following expressions:

$$\alpha(t) = P_{A_{min}} + \triangle_\alpha \qquad \beta(t) = P_{B_{min}} + \triangle_\beta$$

  where $\triangle_\alpha \sim N(P_{A_{max}} - P_{A_{min}} + 1, 1)$, $\triangle_\beta \sim N(P_{B_{max}} - P_{B_{min}} + 1, 1)$, $\alpha(t)$ and $\beta(t)$ are prices sent at the moment t by the Seller and the Buyer respectively. In a similar manner, volume is normally distributed within the limits, that are defined as follow: $V_{min} = V_{A_{min}} = V_{B_{min}}$ and $V_{max} = V_{A_{max}} = V_{B_{max}}$.

- *Strongly calibrated agents* pick a price generated with two major parameters. One parameter γ reproduces series' tendency, for instance, slow decay from maximum to minimum price during one day. The other parameter δ, normally distributed in $N(0,1)$, delivers a generic price fluctuation. An example of price formation, characterized by a slow decay, can be described as follow:

$$\gamma(0) = 1 \qquad \delta(t) \sim N(0,1)$$

$$\gamma(t) = \gamma(t-1) - 0.001 \times t$$

$$P(t) = P_{min} + (P_{max} - P_{min} + 1) \times \gamma(t) \times \delta(t)$$

  More complex price dynamics require a modification of γ parameter description. Volume, as in the previous cases, is a normally distributed value within the given range.

Using this environment and behavior calibration, we show that Uncalibrated agents fail at delivering both realistic prices and quantitative stylized facts. Thus a minimum level of calibration is necessary in the system, which directly question the fact that "zero is enough". In other terms, we show that agents actually should have non-zero intelligence, in order to perform results that are qualitatively and quantitatively congruent with empirically observed motions in real stock markets.

4.3 One single stock detailed results

This section demonstrates the simulations using BNP PARIBAS intraday price series 1^{st}*August* 2002 as a benchmark. The following list sums up the initial settings in the simulations: $P_{min} = 46.25$, $P_{max} = 48$, $P_{mean} = 47.26$, $P_{sd} = 0.35$, $V_{min} = 1$, $V_{max} = 54000$, $V_{mean} = 876.05$, $V_{sd} = 2016.19$, $Number\ of\ fixed\ prices = 4000$. Their applications for each calibration method are described in the following items:

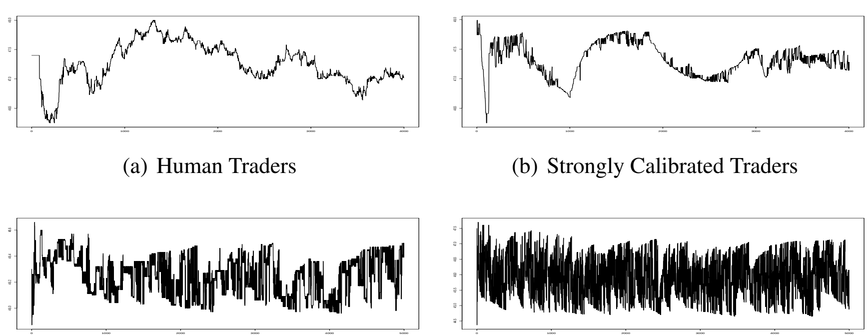
- *Uncalibrated agents* pick a log-normally distributed price $\alpha(t) \sim Log-N(47.26, 0.35)$ and volume $v(t) = Log-N(876.05, 2016.19)$ and randomly set transaction direction (buy/sell). There are three order types, the possibility to send a limit order is 80%, market order - 15%, cancel order - 5%.
- *Statistically calibrated agents* pick a normally distributed price within the ranges: $[45, 48]$ for Ask order and $[46.25, 50]$ for Bid. Transaction volume is normally distributed value between 1 and 54000. These ranges correspond to reality. These agents send the same proportion of order types as the uncalibrated agents.
- *Strongly calibrated agents* pick a price generated by the two parameters evoked previously that provide a price tendency and generic fluctuations. In addition, if we need to get three extreme maximum points in the price series, parameter γ should be coded as illustrated in the Algorithm 1.

This code defines the curve dynamics and stick price from tour to tour (tick by tick). The fluctuation, typical for real market, is provided by the parameter $\delta(t) \sim$

if $t < 50$ **then**
  | $\gamma = 1 - t \times 0.02$
else if $t >= 50$ **and** $t < 100$ **then**
  | $\gamma = 1.2 - t \times 0.01$
else if $t >= 100$ **and** $t < 200$ **then**
  | $\gamma = 0.7 - t \times 0.001$
else if $t >= 200$ **then**
  | $\gamma = 1 - t \times 0.0001$

Algorithm 1: Strong calibration

$N(0,1)$. Finally, the price is defined as $P(t) = 46.25 + (48 - 46.25 + 1) \times \gamma(t) \times \delta(t)$. The volume is defined as in the statistical calibration case. Order types proportion remains the same. In these experiments we use 20 ”big fishes”, that set the orders with volume from 1 to 54000 units, and 100 ”small fishes” agents, that set the orders with volume from 1 to 54 units. Figure 1 shows the price plot, performed by different types of agents. As one can notice, no one can confuse price series of uncalibrated



(a) Human Traders

(b) Strongly Calibrated Traders

(c) Statisticaly Calibrated Traders

(d) Uncalibrated Traders

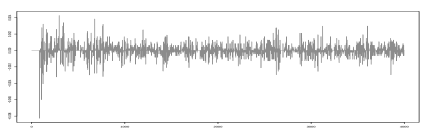
Fig. 1: Intraday price series

agent (even with budget constrains) with real world data. We use a Wilcoxon non-parametric test to estimate the difference in the median of generated data. According to the $p - value = 2.2e - 16$ of the Wilcoxon Test hypothesis (H_0 : ATOM generated prices series are similar to real data) the Null should be rejected. Nevertheless, our target is not to exactly duplicate real price series, but to obtain a not-so-high volatile price series, with statistical characteristics close to the real ones. Table 1 present the statistical properties of this experiment. Quantitative characteristics coming from uncalibrated agents are far from real one. As expected, prices performed by strongly calibrated agent are much more realistic.

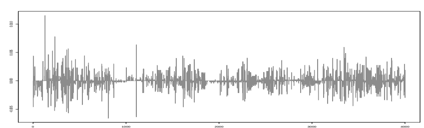
	Real	Strong	Statistical	Uncalibrated
Mean	47.2550	47.3195	46.2435	45.9331
Median	47.2300	47.3200	46.2600	45.9400
Variance	0.1218	0.0771	0.0198	0.4262
Stdev	0.3491	0.2777	0.1407	0.6529
Skewness	-0.2885	-0.3099	-0.0247	0.0108
Kurtosis	-0.0148	-0.0028	-0.8709	-0.9180

Table 1: Basic statistics for price series

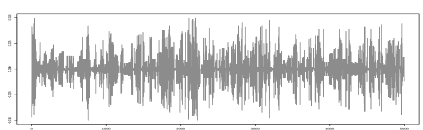
Quantitative properties of ATOM generated price series are far from being real. We now move to consider the properties of the return series (figure 2). According to $p - value_{strong} = 0.9751$, $p - value_{statistical} = 0.644$, $p - value_{uncalibrated} = 0.4138$ of Wilcoxon Test, the series generated by the strongly calibrated agents are close to the real one.



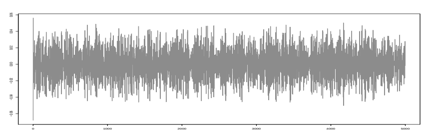
(a) Human Traders



(b) Strongly Calibrated Traders



(c) Statisticaly Calibrated Traders



(d) Uncalibrated Traders

Fig. 2: Intraday return

Even if uncalibrated agents are able to reproduce some of the main stylized facts such as the non Gaussian return distribution (in all cases the Shapiro-Wilk test rejects the Normality hypothesis), positive autocorrelation in absolute returns (Ljung-Box test allows for the rejection of the Null "independence in a given time series"), slow decay of autocorrelation in absolute returns, the corresponding quantitative characteristics do not fit real ones (see table 2).

Uncalibrated agents' returns show high variance and standard deviation comparing with other time series, and at the same time, a very low level of kurtosis (which is not typical for real market data), pretty close to normal one, hence high pick is not really observed in return distribution of uncalibrated agents' series. Table 2 reports the facts, that even the lack of specific strategies with simple calibration mechanisms may provide an approach of quantitative characteristics to the real ones.

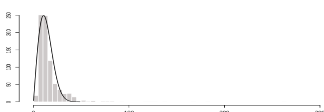
	Real	Strong	Statistical	Uncalibrated
Mean	-0.000002	-0.000004	-0.0000006	-0.000006
Variance	0.000000	0.000001	0.000006	0.000255
Stdev	0.000573	0.000993	0.002377	0.015963
Skewness	-0.974680	0.462673	-0.003997	0.007360
Kurtosis	21.324362	12.751463	2.987817	0.574427

Table 2: Basic statistics of return

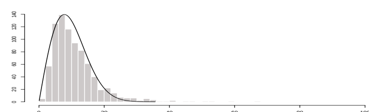
4.4 Population statistics

We now consider a population statistics or sampling distribution approach to investigate the return properties delivered by ATOM. This technique is not widely used in current financial data analysis. One example of such approach can be found in [14], who demonstrate that the sampling distribution of mean values is $Student-t$, standard deviations - χ^2 and kurtosis - Weilbull distribution.

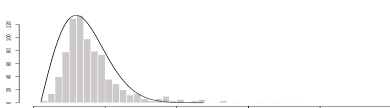
In this research, we show that even if the statistics calculated with the ATOM generated data differ from real-world ones, once considered as population statistics, they follow the similar distributions with different parametrization (see figures 3, 4, 5). For this experiment, 37 assets per 22 days (a total of 814 intraday trading series) were modeled using calibration approaches described in the section 4.2.



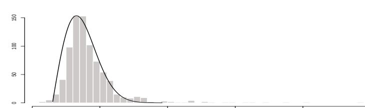
(a) Human Traders



(b) Strongly Calibrated Traders

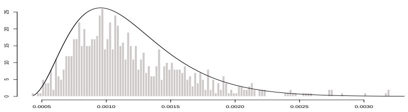
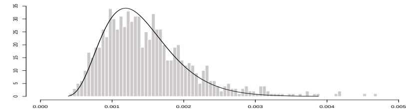


(c) Statisticaly Calibrated Traders

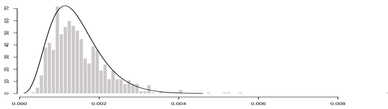
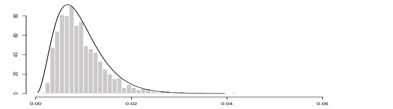


(d) Uncalibrated Traders

Fig. 3: Histogram - distribution of kurtosis; Solid curve - Weilbull distribution

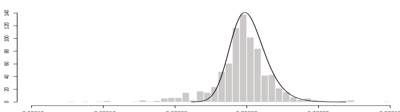
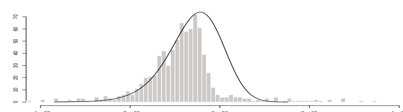



(a) Human Traders (b) Strongly Calibrated Traders

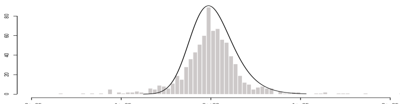
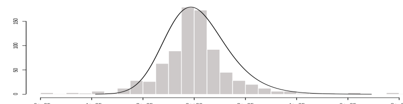



(c) Statisticaly Calibrated Traders (d) Uncalibrated Traders

Fig. 4: Histogram - distribution of standard deviations; Solid curve - χ^2 distribution

(a) Human Traders (b) Strongly Calibrated Traders

(c) Statisticaly Calibrated Traders (d) Uncalibrated Traders

Fig. 5: Histogram - distribution of mean values; Solid curve - $Student-t$ distribution

Conclusion

In this paper we first present our simulation environment, that allows any kind of agents behavior and artificial intelligence specification. This platform can easily be calibrated to match specific features judged as central with regards to a given real-world stock market. In this article, we use this calibration facility to investigate the following question: "What is the minimal level of artificial agents intelligence to get simulated, realistic market prices ?" Based on the simulations, we show that there are significant number of important features of real markets that are not sufficiently delivered by basic artificial intelligence designs. Only with a series of specifications concerning agents' behavior realistic quantitative stylized facts can be obtained. Among other results, we show that *strongly calibrated agents* are definitely much less complex than human beings, and much more complex than uncalibrated (ZIT) ones. Nevertheless, results, performed by *strongly calibrated agents*, are in qualitative and quantitative agreement with empirically observed behavior of prices on real stock markets. We therefore discuss the *"zero is enough"* result that states that sophisticated behaviors are useless to understand how market motions emerge, even at the intraday level. We present an extensive empirical analysis to support this statements. From a practical point of view, this research suggests that if one wants to conduct policy-oriented experiments focusing on technical features of the market-microstructure (for example, to investigate the impact of the tick size on market liquidity and volatility), a minimal calibration of agents population is clearly necessary. Although this calibration is clearly necessary with regard to the desired properties of return series generated with agents population, a special attention should also be put on the price motions themselves.

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