A Multi-Agent Resource Negotiation for the Utilitarian Social Welfare

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Abstract. The multi-agent resource allocation problem corresponds to the negotiation of m resources among n autonomous agents, in order to maximize a social welfare function. Contrary to some former studies, the purpose is neither here to simply determine a socially optimal resource allocation nor to prove the existence of a transaction sequence leading to this optimum, but to find a practical transaction sequence among agents, for any type of contact networks. With this intention, we study various agent behaviors in order to identify which one leads the most often to an optimal resource allocation. The allocation that is reached, can be viewed as an emergent phenomenon that comes out from local interactions among the agents.
After a study of different transaction types, we show that, among the set of studied transactions, the so called "social gift" transaction, is the most efficient one for solving the resource allocation problem associated with the utilitarian social welfare.

1 Introduction

The multi-agent resource allocation problem, which is at the interface of Computer Science and Economics, has been studied for a long time, either within a centralized or a distributed allocation framework. In the studies with a centralized approach, the agents report their preferences on the resources to an auctioneer, which then determines the final resource allocation. Within this context, authors [2, 11] have suggested different transaction models for given types of auctions. In the studies with a distributed approach, the initial resource allocation evolves by means of local negotiations among the agents.

An optimal allocation is sometimes an ashamed notion in the literature. Let us recall the definitions of the solutions of interest.

Global optimum: A resource allocation is a global optimum if there does not exist any other resource allocation with a better social value. A global optimum is independent of the types of transactions that are allowed among the agents.

Moreover, the social value is unique but several resource allocations can correspond to it. However, depending on the initial allocation or on the allowed transaction types, this optimum may not be reachable.

T-global optimum: A resource allocation is a T-global optimum if there does not exist any sequence of transactions belonging to the set of transactions T that allow reaching a resource allocation with a greater social welfare value. Such an allocation is most of the time suboptimal.

A first set of studies focuses on the mathematical properties related to the types of considered transactions. A classification of the basic transactions has been established along with theorems on the existence or the non-existence of a specific transaction sequence, from any initial resource allocation to a global optimum in [10]. However, these studies do not exhibit any mechanism that can be used to reach the optimal resource allocation: They only proved its existence. Along the same lines, mathematical properties on some classes of utility functions and payment functions have been studied in [5] in order to design mechanisms, which terminates after a finite number of iterations. In [6], the authors study the acceptability criterion and the transaction properties, but do not provide any explicit mechanism.

In a second set of studies, the authors defined new agent behaviors. Some of them have identified conditions favoring equitable deals [7] and others have studied envy-freeness in the resource allocation process [3, 4]. None of these studies can exhibit a sequence of acceptable transactions (i.e., that satisfy the criteria imposed by the agents) from an initial resource allocation to a T-global or a global optimum. In addition, no comparison was made between the social value of the resource allocation reached at the end of the negotiation process and the globally optimal social value.

In this study, our purpose is to design a negotiation mechanism which is able to converge, in practice, either towards a global optimum, or towards a near optimal solution. Section 2 defines the transactions that are used in this study, and discusses the convergence issues of the negotiation process. Section 3 details the experiment protocol and the evaluation criteria of the mechanisms. Finally, Sect.4 investigates further the social gift transaction, and the impact of the agent behavior in a negotiation process based on such transactions.

1.1 Multi-agent resource allocation problem

The multi-agent resource allocation problem is defined by a set of autonomous agents, that are able to locally negotiate their resources. Let us consider a multi-agent system where $\mathcal{R} = \{r_1, \ldots, r_m\}$ is the finite set of available resources. We assume the resources to be initially distributed over a population of n agents: $\mathcal{A} = \{a_1, \ldots, a_n\}$. Each agent owns a set of resources, denoted by $\mathcal{R}_a$. The preferences of the agents are represented by a utility function: $u_a : \mathcal{R} \rightarrow \mathbb{R}$. A resource allocation o is a partitioning of all the resources among the agents, and can be expressed using the resource set of each agent: $o = [\{\mathcal{R}_1\}, \ldots, \{\mathcal{R}_n\}]$. Let $\mathcal{O}$ be the set of all possible allocations.

The usual definition of a transaction is the following one: A transaction $\delta = (o, o')$ is a pair of resource allocations, where o and o' define the state of the multi-agent system respectively before and after a given negotiation involving a given subset of agents. In practice, an agent does not have a global view of the system. This is the reason why, in our study, we consider that initially, agents only know their preferences and their neighbor list. This implies that transactions are based on local information only. Let $\mathcal{R}_{a \leftrightarrow a'}$ be the set of involved resources during a transaction between agents a and a'.

Definition 1 (Transaction). *A transaction, initiated by an agent a and in which agents $a', a'', \ldots$ are involved, is a list of resource sets that are exchanged between the agent initiator and the involved agents.*

$$\delta_a = [\mathcal{R}_{a \leftrightarrow a'}, \mathcal{R}_{a \leftrightarrow a''}, \ldots]. \tag{1}$$

In our study, we focus on a homogeneous agent society, in which resources are assumed discrete, not shareable, not divisible, not consumable and unique. Hence, the resources cannot be modified by the agents, but only exchanged during the negotiation process.

1.2 Contact Network

The contact network represents the graph of the relationships among the agents: Each agent has a list of neighbors with whom he is able to communicate. Most of the studies rely on the hypothesis of a complete contact network. An agent is then able to negotiate with any other agent in a multi-agent system: This is strongly impacting the resource allocation process.

However, this hypothesis is not realistic as soon as real world applications are considered. For instance, in the case of social networks, a person only knows a subset of the overall set of actors in the network. In this study, we consider that the contact network can be any connected graph, ranging from a complete graph to a small-world.

According to the allowed transaction types, a mechanism which converges towards an optimal resource allocation in the case of a complete contact network, may only converge towards a sub-optimal resource allocation in the case of a restricted contact network. The mean connectivity degree of a contact network is defined in this study as the average number of neighbors of an agent.

1.3 Social welfare

Social welfare functions [1, 9] are usually used in order to evaluate a multi-agent system like a whole throughout the welfare evaluation of each agent in the system.

Definition 2 (Utilitarian social welfare). *The utilitarian social welfare, denoted by sw_u, is defined as the summation of the agent welfare. For a given resource allocation o:*

$$sw_u(o) = \sum_{a \in \mathcal{A}} u_a(\mathcal{R}_a) = \sum_{a \in \mathcal{A}} \sum_{r \in \mathcal{R}_a} u_a(r). \tag{2}$$

Definition 3 (Egalitarian social welfare). *The egalitarian social welfare, denoted by sw_e, is defined by the utility of the poorest agent. For a given resource allocation o:*

$$sw_e(o) = \min_{a\in\mathcal{A}} u_a(\mathcal{R}_a) = \min_{a\in\mathcal{A}} \sum_{r\in\mathcal{R}_a} u_a(r). \tag{3}$$

The purpose of our study is to design practical mechanisms of resource negotiations among agents, which guarantee that the negotiations end after a finite number of steps, with a final resource allocation that is as close as possible to the optimal social value, for any arbitrary connected contact network.

2 Transaction

In a multi-agent resource allocation problem, the compensatory payments are usually allowed during the negotiation process. Allowing the compensatory payments, from an agent's point of view, corresponds to an extension of the acceptable transaction set. However, even if the use of money is constrained (no money creation during a transaction), there is often no limit on agent budgets in order to perform the transactions in most published studies. Questions related to compensatory payments are beyond the scope of our study: They are not studied in the sequel. The agent preferences are expressed by means of k-additive utility functions [8], with positive utilities. Moreover, our study is restricted to the most widely used transaction family: Bilateral transactions in which only two agents at a time can be involved.

2.1 Convergence

In [10], it has been proved that there always exists a sequence of non rational "original contracts" leading, from any initial resource allocation, to a global optimum. A transaction is rational when each agent involved in it increases its utility, and an "original contract" corresponds to the purchase of a resource. The existence of a such sequence does not guarantee that the negotiation process ends with a socially optimal resource allocation. Indeed, since the agents are not rational, they are not able to distinguish profitable deals from non profitable ones.

2.2 Acceptability criteria

In order to negotiate in an appropriate way, acceptability criteria are usually enforced with respect to the agent behavior. They restrict a lot the set of acceptable transactions. The negotiation process ends when no agent is able to find an acceptable transaction.

Let us assume that, at a given time, agent a initiates a transaction $\delta(o, o')$ with agent a', resulting of an evolution of the resource allocation $o = [\ldots\{\mathcal{R}_i\}\ldots\{\mathcal{R}_j\}\ldots]$ towards a new one $o' = [\ldots\{\mathcal{R}'_i\}\ldots\{\mathcal{R}'_j\}\ldots]$.

Definition 4 (Rational agent). *A rational agent is an agent who only accepts transactions that increase his utility. If agent a is rational, he will accept a transaction only if:*

$$u_a(\mathcal{R}'_a) > u_a(\mathcal{R}_a).$$

The rationality criterion is the most widely used in the literature, especially in the case of non cooperative selfish agents.

Definition 5 (Rational transaction). *A rational transaction is a transaction in which all involved agents are rational. If a transaction is rational, involved agents accept it if:*

$$u_a(\mathcal{R}'_a) > u_a(\mathcal{R}_a) \quad \textit{and} \quad u_{a'}(\mathcal{R}'_{a'}) > u_{a'}(\mathcal{R}_{a'}).$$

Proposition 6. *A multi-agent resource allocation process that uses rational transactions ends after a finite number of transactions.*

However, the restrictions imposed by the rationality criterion to the set of possible acceptable transactions may lead to a sub-optimal resource allocation at the end of the negotiation process.

Another criterion that ensures the end of the resource allocation process after a finite number of transactions is the sociability. That criterion is based on a local evaluation of the social welfare.

Definition 7 (Social agent). *A social agent is an agent who can only accept transactions that increase the social welfare function of the agent system.*

Definition 8 (Social transaction). *A transaction is social if the value of the social welfare function considered increases. Such a transaction can only be accepted by the involved agents if:*

$$sw_U(o') > sw_U(o) \quad o, o' \in \mathcal{O} \textit{ such that } o \xrightarrow{\delta} o'.$$

In order to determine the value associated with the social welfare function, it is essential to have a global knowledge of the multi-agent system state: The utility of each agent is used to compute the social value. However, it is possible to determine the variation of this value based on local information only: It is then not necessary to determine its value:

$$\begin{aligned} & sw_u(o') > sw_u(o) \\ \Rightarrow \quad & \sum_{a \in \mathcal{A}} u_a(\mathcal{R}'_a) > \sum_{a \in \mathcal{A}} u_a(\mathcal{R}_a) \\ \Rightarrow \quad & u_a(\mathcal{R}'_a) + u_{a'}(\mathcal{R}'_{a'}) > u_a(\mathcal{R}_a) + u_{a'}(\mathcal{R}_{a'}). \end{aligned}$$

Indeed, since only two agents are involved in the current transaction, only their resource bundle change. Then, the utility of the agents that are not involved in the transaction can be considered as a constant value. Let us note that a rational transaction is always social, whereas the opposite is not true.

2.3 Transaction type

We distinguish below three bilateral types of transactions. Others are combination of these basic transaction types. In order to illustrate them, let us consider the case where an agent initiator a negotiates with an agent a'. Each of them owns m_a and $m_{a'}$ resources respectively.

First, *the gift*. This is a transaction during which the initiator gives one of his resources to the involved agent. The gift transaction transaction, which is the simplest possible one, cannot be rational for the initiator and is always rational for the agent participant (since utilities are positive).

Then, *the swap*. It is a transaction where each agent exchanges a unique resource. This transaction is symmetric, i.e., an agent that initially owns m_a resources, will have the same number of resources at the end of the allocation process. Hence, a global optimum can be reached only if the initial resource allocation has the same resource distribution as one of the optimal resource allocations. The total number of swaps between a and a' is $m_a \times m_{a'}$.

Finally, *the cluster-swap* (CS) is a transaction during which the agents can involve a subset of resources. This transaction can be asymmetric. The swap is a particular case where both agents involve only one resource each. The number of possible cluster-swaps for a transaction initiated by the agent a with a' is $(2^{m_a} - 1) \times (2^{m_{a'}} - 1)$, i.e., we do not allow cluster swaps where one of the agents does not give any of his resources.

When combining these three types of transactions with the acceptability criteria, the following transactions can be defined:

1. the social gift,
2. the social swap,
3. the rational swap,
4. the social cluster-swap,
5. the rational cluster-swap.

2.4 Communication protocol

In order to compare and evaluate the different types of transactions, we develop a multi-agent system with sequential negotiations: Only one agent at a time is able to negotiate. Note that if parallel transactions were performed, except maybe for very specific synchronisation rules, it would only affect the convergence speed but not the quality of the final allocation.

The agent initiator is randomly chosen in the multi-agent system. Agents accept or refuse transactions according to their own criterion. The negotiation process ends when no agent is able to find an acceptable transaction in his neighborhood.

The communication protocol is described in Fig.1. In the specific case of gift transactions, during which only the agent initiator gives a resource without counterpart, the dashed part should be omitted. When the agent initiator a selects and offers a resource r, the involved agent a' has to report the utility that he associates with a resource r and offers a resource r'. Then, the agent initiator determines whether or not the transaction is acceptable. He decides to perform the transaction if the acceptability criterion are satisfied for both agents, or he has to determine who has to change his offer and then suggest another resource. For instance, with a utilitarian social welfare function, the test can be on the comparison of what agents give and what they receive: $u_a(r') - u_a(r') >^? u_{a'}(r) - u_{a'}(r')$. If no agent is able to suggest a different resource, the negotiation then ends.

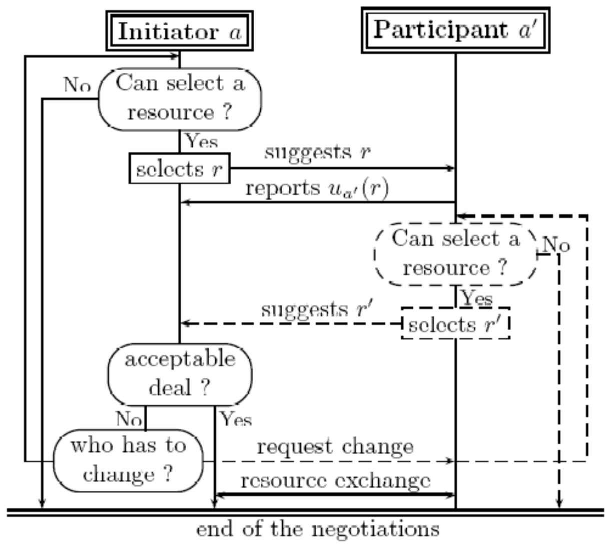

Fig. 1. Communication protocol

3 Experiments

3.1 Experiment protocol

The experiments have been done on multi-agent systems of various sizes. For each of them, different types of contact network have been created, some complete and some random with a mean connectivity degree of $n/2$. For each setting, a large number of multi-agent systems has been generated, and in each case, 100 instances have been run using different initial resource allocations.

For each negotiation process, the agent initiator is chosen randomly. He always sorts his bundle of resources according to his utility function: Even if agents are not rational, they try to give first their resources associated with the lowest utility. The default behavior of the agents is the negotiation with one selected neighbor, in order to find an acceptable deal according to the acceptability criterion in use.

3.2 Evaluation criteria

An evaluation protocol has been designed in order to compare the transactions that are used during the negotiations. Various criteria have been considered.

Number of performed transactions: It is the overall number of transactions that are performed during the negotiation process. Negotiations using restrictive transactions, such as rational transactions, will stop faster than negotiations using more permissive transactions, such as social transactions.

Number of exchanged resources: Some transactions, such as the cluster-swap, tolerate that an agent involves more than one resource whereas others prohibit that, such as the gift. One cluster-swap is equivalent to a sequence of, at least, two gifts.

Number of speech turns: It corresponds to the number of negotiation initializations. If associated with the number of performed transactions, the number of aborted negotiations can be deducted.

Number of attempted transactions: Depending on the agent behavior, it could be more or less difficult to find an acceptable deal. This measure gives an estimation of the negotiation length.

In addition to these criteria, we evaluate the gap between the optimal social value and the social value associated with the resource allocation reached at the end of the negotiation process.

3.3 Optimal value determination

The optimal social value associated with a resource allocation instance can be determined by means of a $0-1$ linear program. Denote by $\mathcal{A}$ the finite set of agents and by $\mathcal{R}$ the whole set of available resources in the multi-agent system. The variables of this $0-1$ linear program are $x_{ra} \in \{0,1\}$ for $r \in \mathcal{R}$ and $a \in \mathcal{A}$:

$$x_{ra} = \begin{cases} 1 \text{ if the agent } a \text{ owns the resource } r \\ 0 \text{ otherwise.} \end{cases}$$

Utilitarian case The $0-1$ linear program that corresponds to the maximization of the utilitarian social welfare, which can be written as:

$$sw_u^\star = \begin{cases} \max \sum_{a \in \mathcal{A}} \sum_{r \in \mathcal{R}} u_a(r) x_{ra} & \\ \text{subject to: } \sum_{a \in \mathcal{A}} x_{ra} = 1 & r \in \mathcal{R} \\ \qquad\qquad x_{ra} \in \{0,1\} & r \in \mathcal{R}, a \in \mathcal{A}. \end{cases}$$

Egalitarian case The $0-1$ linear program that corresponds to the maximization of the egalitarian social welfare, which can be written as:

$$sw_e^\star = \begin{cases} \max \min_{a \in \mathcal{A}} \sum_{r \in \mathcal{R}} u_a(r) x_{ra} & \\ \text{subject to: } \sum_{a \in \mathcal{A}} x_{ra} = 1 & r \in \mathcal{R} \\ \qquad\qquad x_{ra} \in \{0,1\} & r \in \mathcal{R}, a \in \mathcal{A}. \end{cases}$$

It is also possible to constraint these models in order to determine the best social value associated with a rational resource allocation. The addition of the following set of constraints is then required:

$$\sum_{r \in \mathcal{R}} u_a(r) x_{ra} \geq u_a^{init} \quad a \in \mathcal{A}$$

where u_a^{init} is the initial utility of the agent a.

3.4 Utilitarian efficiency of the transactions

A summary of all the experiments are presented in this section. First, the results related to a complete contact network are presented, then the results related to a random contact network with a mean connectivity degree of $n/2$. The size of the instances are denoted by $n - m$ where n is the number of agents and m the total number of resources that are uniformly distributed at the outset.

The results obtained with a complete contact network are summarized in Table 1.

Table 1. Gap(%) on a complete contact network

n-m	Social			Rational	
	Gift	Swap	CS	Swap	CS
50-500	0	0.94	0.96	2.15	6.71
100-1000	0	0.76	0.76	1.53	4.9
150-1500	0	0.65	0.71	1.31	3.9
200-2000	0	0.56	0.60	1.15	2.5

The social gift is the lone transaction that is always associated with a convergence towards a global optimum. Even if, in all our experiments, a global optimum is never reached with the other types of transactions, the gap is relatively small: It is thus possible to reach a resource allocation that is socially close to the social value of the global optimum. The negotiation processes that use rational transactions stop further of the optimal social value than the ones that use social transactions as a consequence of a more restrictive criterion. The size of the instances does not seem to have not a strong impact on the quality of the final allocation.

Figure 2 shows the number of performed transactions according to the instance sizes and the types of allowed transactions. The transaction sequences are shorter when rational agents negotiate. The social criterion is more flexible, thus more transactions can be performed by social agents. The number of exchanged resources is greater in the case of social agents than with rational agents, however the difference is not significant as shown in Fig.3. Figure 4 describes the evolution of the number of speech turns: Only a weak variation can be noticed

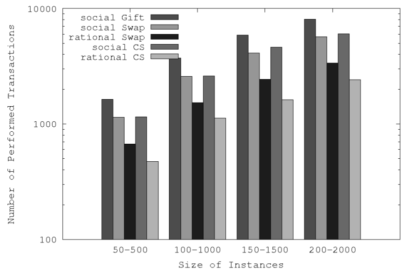

Fig. 2. Behavior Comparison on the number of performed transactions

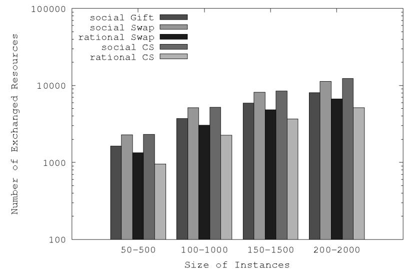

Fig. 3. Behavior comparison on the number of exchanged resources

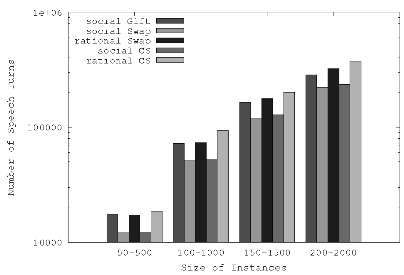

Fig. 4. Behavior Comparison on the number of speech turns

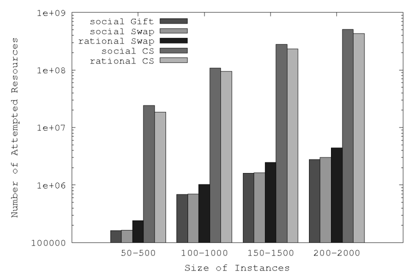

Fig. 5. Behavior comparison on the number of attempted transactions

depending on of the transaction type used. Finally, Fig.5 shows the number of attempted transactions. One can notice that using cluster-swap transactions leads to a very large number of attempted transactions.

Results with a random contact network are shown in Table 2. The contact network itself has a large impact on the quality of the final allocation. Depending on the used transactions, the network limits the resource traffic. During the experiments, the global optimum is seldom reached. The smallest gap is always obtained by the social gift. The negotiation process ends on socially weaker allocations if restrictive transactions are used. However, the weaker the connectivity

Table 2. Gap(%) on a random contact network

n-m	Social			Rational	
	Gift	Swap	CS	Swap	CS
50-500	1.3	3.41	3.4	6.05	5.88
100-1000	0.73	1.88	1.72	3.63	3.59
150-1500	0.43	1.3	1.35	2.69	2.42
200-2000	0.31	1.22	1.02	2.3	2.05

of the contact network is, the larger the gap is. It is a similar behavior for the standard deviation, which is larger than with a complete contact network.

4 Social Gift

4.1 Behavior variants

The behavior of the agents has an important impact on the quality of the resource allocation that is finally reached. In order to study further the influence of the agent behavior, the social gift is used on a complete contact network. If the agent initiator and the selected neighbor find an acceptable transaction, they perform this transaction. In a case of a refusal, three different options are possible for the agent initiator:

1. abort the negotiation
2. choose another resource with the same neighbor
3. choose another neighbor with the same resource.

Based on this option set, four different behaviors (described below) can be defined. For each behavior, the initiator a gives a unique resource according to the definition of the gift in Sect.2.3. After the identification of an acceptable deal or the end of the negotiation, a new initiator is randomly chosen.

First, behavior "A" is described in Table 3. The agent initiator a selects randomly a neighbor and tries to give the resource associated with the lowest utility. If this is not an acceptable transaction, then the agent initiator aborts the negotiation.

If the agent initiator a adopts behavior "B" described in Table 4, then he selects randomly a neighbor and negotiates his resources, starting with the one associated with the lowest utility and increasing it gradually. If, no resource can constitute an acceptable transaction, then the negotiation stops.

Table 3. Behavior "A"

Sort my resource bundle
Select my lowest utility resource r
Randomly select a neighbor a'
If $u_a(r) < u_{a'}(r)$
give r to a'
Abort the negotiation

Table 4. Behavior "B"

Sort my resource bundle
Randomly select a neighbor a'
For each resource r of my bundle
If $u_a(r) < u_{a'}(r)$
give r to a'
break
Abort the negotiation

Table 5 describes agent behavior "C". The agent initiator a negotiates only his lowest utility resource with all the agents of his neighborhood. In order to avoid a bias due to the sequential selection of the selected neighbor, a random permutation is applied on the neighbor list of the initiator. If no agent assigns a greater utility to the resource than the initiator, then the negotiation aborts.

Finally, with behavior “D” described in Table 6, the agent initiator negotiates every resource as for agent behavior “C” one after the other, with all his neighbors for each of them. The same technique is used in order to avoid the bias due to a sequential selection of the selected neighbor. After the negotiation of all his resources with all his neighbors, the agent initiator aborts the negotiation.

Table 5. Behavior “C”

```
Sort my resource bundle
Select my lowest utility resource r
  For each neighbor a'
    If u_a(r) < u_a'(r)
      give r to a'
      break
Abort the negotiation
```

Table 6. Behavior “D”

```
Sort my resource bundle
For each resource r of my bundle
  For each neighbor a'
    If u_a(r) < u_a'(r)
      give r to a'
      break
Abort the negotiation
```

4.2 Behavior efficiency

The four behaviors defined in Sect.4.1 have been evaluated using the criterion defined in Sect.3.2. Experiments have been conducted on a complete contact network. The experiment protocol is as described in Sect.3.1. In all our experiments, behaviors “A” and “C” have never been able to reach a socially optimal resource allocation. However, the gap between the reached allocation and the global optimum remains always less than 2.15%. The mean deviation is small: Less than 0.2% in all cases. Independently of the initial resource allocation, the allocations that are reached at the end of the negotiations have very close social values.

Table 7. Social gap (%) comparison of the behaviors

n-m	A	B	C	D
50-500	1.2	0	1.1	0
100-1000	0.5	0	0.5	0
150-1500	0.3	0	0.3	0
200-2000	0.2	0	0.2	0

Behaviors “B” and “D” always stop the negotiation process on a global optimum. In practice, their results are identical, however, in theory, the convergence towards the global optimum is only guaranteed in the case of behavior “D”. It is always possible to design a instance where the usage of behavior “B” cannot reach the global optimum. However, such a guarantee has a cost as shown by the behavior comparison in terms of performed transactions (Fig.6), speech turns (Fig.7), and attempted transactions (Fig.8).

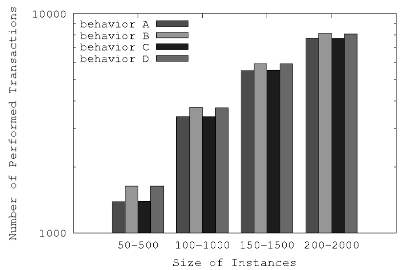

Fig. 6. Behavior Comparison on the number of performed transactions

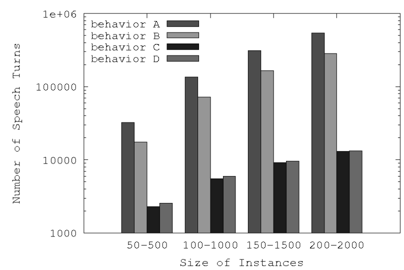

Fig. 7. Behavior comparison on the number of speech turns

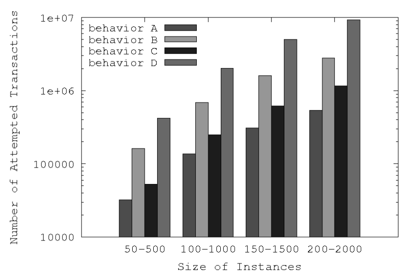

Fig. 8. Behavior comparison on the number of attempted transactions

As shown on Fig.6, the number of performed transactions does not vary appreciably from one behavior to the next. However, Fig.7 shows that the number of speech turns is really higher with behaviors “A” and “B”. Indeed, these last two behaviors are focused on the negotiation with one agent whereas the other ones can change the involved neighbor and therefore can benefit from the neighbor list. Let us note, on Fig.8, that behavior “D” is more expensive in terms of attempted transactions. Negotiation processes among agents that use behavior “D” take more time than others.

Hence, the use of behavior “B” is more interesting than the use of behavior “D”: While behavior “B” leads to the same results in practice than behavior “D”, behavior “D” is more time consuming. However, the efficiency of behavior “D” can be proved whereas it is not possible with behavior “B”.

4.3 Proof of convergence

We now focus on behavior "D", the lone behavior for which it is possible to guarantee the end of the negotiation process on a socially optimal resource allocation. Let us introduce the allocation graph $\mathcal{G}$: Each node of $\mathcal{G}$ represents a resource allocation, and a directed link $\delta(o, o')$ between two nodes o and o' exists if an acceptable transaction δ which changes o in o' exists.

An outgoing link δ of a node o corresponds to an acceptable transaction that changes the resource allocation o into another resource allocation o'. An incoming link δ to a node o corresponds to an acceptable transaction that changes a given initial resource allocation o' into o.

Proposition 9 (Optimum T-global). *The resource allocation corresponding to a node without outgoing link (i.e., a "sink") and at least one incoming link is an optimum T-global.*

Proposition 10 (Global optimum). *A global optimum is a node without outgoing link which represents a resource allocation associated with a maximal social value.*

If the current resource allocation is a social optimum, no acceptable transaction can be performed to improve the social welfare value, meaning no outgoing link exists.

Theorem 11. *Consider a set of resources which are discrete, not shareable, static and unique. The negotiation process of a multi-agent resource allocation instance based on such resources and on a complete contact network converge towards the global optimum using social gifts.*

Proof. A resource allocation, which corresponds to a utilitarian global optimum is such that each resource is distributed to an agent who assigns the greatest utility to it.

Since the contact network is complete, an agent can always initiate a social gift with any other agent, which associates a greater utility to the involved resource. If it is not possible, then the current resource allocation is already a global optimum. Hence, it always exists a sequence of social gift transactions that assigns a resource to an agent who assigns the greatest utility for it. The allocation graph $\mathcal{G}$ is connected: It is possible from any initial node to find a sequence of social gifts leading to an optimum.

In graph $\mathcal{G}$, a T-global optimum is a resource allocation where no agent can find a social gift improving the social welfare value. Although, if no social gift can be performed, each resource is then assigned to an agent who evaluates it with the greatest utility, which corresponds to the property of a global optimum.

From any initial node of the graph $\mathcal{G}$, agents initiate social gift transactions in order to improve the social welfare value. If there does not exist any outgoing link, that means there is no possible social gift improving the social welfare value: No agent can find another agent who associates a greater utility to one of his resources.

Hence, the current resource allocation is a global optimum.

4.4 Egalitarian efficiency of the social gift

Among the studied transactions in this paper, the social gift appears as the most efficient transaction in a multi-agent resource allocation process, when the utilitarian social welfare is considered. However, the issue of the efficiency of this transaction can be raised when another welfare function is considered.

The aim of the egalitarian social welfare is to maximize the utility of the poorest agent. Thus, the standard deviation among the agent utility decreases during the negotiation process. The social criterion in the case of the egalitarian social welfare can be interpreted as follows: The poorest agent at the end of such a transaction must not be poorer than the poorest agent before the transaction.

The gap between the social value of the resource allocation on which the negotiation process ends and the social value of the global optimum are described in Table 8. The social gift has a high gap, which means that the negotiation

Table 8. Optimality gaps for the social gift with the egalitarian welfare is considered

$n-m$	50-500	100-1000	150-1500	200-2000
Gap(%)	31.08	32.61	31.50	32.4

process ends on a resource allocation which is far from the global optimum.

Example 12. Let us consider a multi-agent system in which the standard deviation among the agent utility is small enough compared to the maximal utility value for a resource. This situation is plausible since the egalitarian social welfare is considered.

Let us assume that agent a owns resource r and one of his neighbor a' owns resource r'. Their preferences are as follows: $u_a(r) = 50$, $u_a(r') = 100$, $u_{a'}(r) = 100$, and $u_{a'}(r') = 50$. In order to increase the social welfare value, these agents have to exchange their resources. None of them is able to initiate a transaction: If an agent gives a resource, he becomes the poorest agent and the egalitarian welfare value decreases. Hence, the resource allocation on which the negotiation process ends can be far from the global optimum.

The social gift is not an efficient transaction when the egalitarian social welfare function is considered. An efficient mechanism should allow other transaction types than the gift one.

5 Conclusion

In this study, we have designed a mechanism, which incites the multi-agent resource allocation process to converge towards a global optimal, or when it is not possible, towards a socially close resource allocation. This mechanism can be used in practice thanks to a distributed approach based on agents and to the introduction of the notion of contact network. Moreover, this mechanism is adaptable: The addition of new agents is possible during the negotiation process without decreasing the quality of the resource allocation that is reached. The mechanism is also an “any-time” algorithm: The quality of the solution increases gradually and the negotiation process can be interrupted anytime.

The described mechanism is efficient for improving the solution of the multi-agent resource allocation problem when the utilitarian social welfare is considered. However, it is not adapted to the egalitarian social welfare. New practical mechanisms have to be designed in that case.

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