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Agent-Based Reallocation Problem on Social Networks

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Abstract Resource reallocation problems aim to determine an allocation maximizing a given objective function. Numerous applications are based on the assumption of restricted contacts between entities but, up to now, studies have been based on unrealistic contexts. Indeed, most of the time, agents are omniscient and/or have complete communication abilities, which are not plausible assumptions in many applications. A solution does not only consist in an optimal allocation, but in a sequence of transactions changing an initial allocation into an optimal solution. We show that the individual rationality does not allow the achievement of socially optimal allocations, and we propose a more suitable criterion: the sociability. Our method provides a sequence of transactions leading to an optimal allocation, with any restriction on agents' communication abilities. Provided solutions can be viewed as emergent phenomena.

Keywords Multiagent resource allocation problems · Negotiation · Simulation · Nash welfare

1 Introduction

Allocation problems can be encountered everywhere in real life through countless applications. However, even if their aim is often to identify a resource distribution maximizing a given objective, different approaches exist. Some solving techniques only focus on the best resource distribution and consider resources from one side and agents on the other side. It is the case for all centralized techniques, like with combinatorial auctions and the so-called winner determination problems (Sandholm 2002).

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However, in many applications, resources are initially already distributed between agents within the population. In such cases, the aim is to identify a sequence of transactions leading from the initial resource allocation to an optimal solution. Then, centralized techniques are not suitable to solve allocation problems in such conditions. In this purpose, methods based on agent negotiations have been developed, where autonomous agents negotiate by pair to identify acceptable transactions. Studies in this field are most of the time theoretical. Some studies established the existence or not of transaction sequence to optimal solutions (Sandholm 1998), while others focused on complexity (Dunne et al. 2005) to bound the length of transaction sequences. Classes of utility functions and payment functions are also studied to design convergent negotiation processes (Chevaleyre et al. 2005). They also study different scenarios corresponding to different preference representation and to different acceptability criteria (Endriss et al. 2006; Chevaleyre et al. 2010). However, these studies did not focus on the mechanism to use in order to get an optimal solution, they characterize properties that may favor the achievement of optimal allocations and design abstract frameworks. However, none of them is able to exhibit acceptable transaction sequences or provide the agent behavior to implement in order to negotiate efficiently in practice.

These studies are not always based on realistic assumptions. Indeed, agents are omniscient: they know everything about all other agents, their resources as well as their preferences. Moreover, communication abilities are implicitly assumed complete. It means that an agent is always able to negotiate with any other agent in the population. These assumptions are not realistic. For instance, in networks, a node is only linked with a very restricted number of neighbors, like in peer-to-peer networks, and is not aware of the whole system. As well, in routing problems, all servers are not interconnected. Solutions provided by methods that do not consider restricted communications may not be achievable in practice.

In this paper, we choose to focus on distributed methods to solve resource reallocation problems in a more realistic context. We design the negotiation settings leading negotiations to optimal solutions using local negotiations between agents. Any restriction on the agents' communication abilities can be considered and information privacy can be handled. Agents only have a local perception of the system. The impact of the communication restrictions are also evaluated and features favoring or penalizing the negotiation efficiency are identified. This paper is organized as follows. Section 2 describes the parameters we consider as well as the issues we face. The importance of considering restrictions on communication is also stated. Section 3 presents our solving approach and Sect. 4 present the evaluation protocol. Finally, Sect. 5 describes our results.

2 Issues on Agent Negotiations

2.1 Key Parameters

According to the different facets that we want to evaluate in a negotiation process, we propose to consider several parameters. The formal definition of agents is based on these parameters. An agent is defined with a **bundle** describing the owned resources,

preferences used to evaluate the agent satisfaction, a **behavior** specifying how agents interact, an **acceptability criterion** on which the agent determines if a deal is profitable, and a **neighborhood** representing the communication possibilities.

Definition 1 (*Agent*) An agent $i \in \mathscr{P}$ is a tuple $\langle \mathscr{R}_i, u_i, \mathscr{N}_i, \mathscr{B}_i, \mathscr{C}_i \rangle$, where $\mathscr{R}_i$ is the set of m_i resources the agent owns, u_i is the utility function (the agent preferences), $\mathscr{N}_i$ is the list of n_i neighbors, $\mathscr{B}_i$ defines the agent behavior according to which the agent negotiates, and $\mathscr{C}_i$ is its acceptability criterion on which are based its decisions.

We consider that every agent initially starts negotiating knowing only its resource bundle, its own preferences and the list of possible partners. We choose to limit the perception and the knowledge of the agents in order to get a more realistic environment. Agents are not omniscient and can at most have information from their neighborhood. They can only negotiate with agents from their neighborhood, which restricts a lot the possible transactions. Such assumptions are more restrictive than the usual ones, but represent a more realistic context than the conditions described in former studies.

2.2 Evaluation of a Negotiation Process

Thanks to Definition 1, different facets of allocation problems can be evaluated. In this paper, we choose to focus on the efficiency of negotiation processes. Since an objective is to identify the negotiation settings leading agent negotiations to optimal solutions, the absolute efficiency must be evaluated. Indeed, comparisons with optimal solutions are essential. The quality of two allocations can be compared thanks to the notions of the social choice theory. Different metrics can be used, but we choose to focus on the four most important.

The most widely used notion to evaluate resource allocations is the utilitarian welfare. This notion is used to maximize the average individual welfare in a population.

Definition 2 (*Utilitarian welfare*) The utilitarian welfare of a resource allocation A, denoted by $sw_u(A)$, corresponds to the sum of individual utilities.

$$sw_u(A) = \sum_{i \in \mathscr{P}} u_i(\mathscr{R}_i), \quad A \in \mathscr{A}.$$

The egalitarian welfare of an allocation corresponds to the individual welfare of the poorest agent in the population. Its maximization tends to reduce inequalities in the population.

Definition 3 (*Egalitarian welfare*) The egalitarian welfare of an allocation A, denoted by $sw_e(A)$, corresponds to the individual utility of the poorest agent.

$$sw_e(A) = \min_{i \in \mathscr{P}} u_i(\mathscr{R}_i), \quad A \in \mathscr{A}.$$

The Nash product considers the welfare of the whole population and reduces the inequalities among agents at the same time. The Nash product can be viewed as a compromise between the utilitarian and the egalitarian welfare. In spite of its qualities, a drawback remains: this notion becomes meaningless if non-positive values are used.

Definition 4 (*Nash product*) The Nash product of an allocation A, denoted by $sw_n(A)$, corresponds to the product of individual utilities.

$$sw_n(A) = \prod_{i \in \mathscr{P}} u_i(\mathscr{R}_i), \quad A \in \mathscr{A}.$$

The elitist welfare only considers the welfare of the richest agent in the population. This notion can be useful in the context of artificial agent societies for instance, where agents have a common objective. This objective must be fulfilled, independently of the agent who achieves it.

Definition 5 (*Elitist welfare*) The elitist welfare of an allocation A, denoted by $sw_{e\ell}(A)$, corresponds to the individual utility of the richest agent in the population.

$$sw_{e\ell}(A) = \max_{i \in \mathscr{P}} u_i(\mathscr{R}_i), \quad A \in \mathscr{A}.$$

For each social welfare notions, the optimal value can be determined or estimated by means of a centralized algorithm, as suggested in Nongaillard and Mathieu (2009b) but will not be detailed here. This optimal value can then be used as reference to determine the absolute efficiency of a negotiation process.

The impact of the social graph topology can also be evaluated, in order to determine the cost of the restrictions on agents communications. Imposing restrictions on agent communications will limit the number of possible transactions and then the resource traffic. It will inevitably impact the negotiation efficiency but constitutes a more realistic environment. Topological features favoring or penalizing the efficiency of negotiation processes can then be identified.

The topological sensitivity should also be evaluated. Indeed, considering different graph topologies of the same class, negotiation processes starting from the same initial allocation can achieve different allocations. The topological sensitivity can be evaluated thanks to the standard deviation among the social values achieved at the end of negotiation processes. A large deviation means that the negotiation process is very sensitive to the graph topology, and thus the quality of the provided allocation significantly varies according to the initial conditions. A low standard deviation means that, independently of the initial owner of the resources, the negotiation process leads to optimal solutions or close solutions.

2.3 The Social Graph: An Important Issue?

Since agents communication abilities are usually not restricted in negotiation problems, it is legitimate to investigate the importance of such a parameter. Negotiation processes, which lead to optimal solutions according to complete communication abilities (i.e. based on complete social graphs), may only lead to solutions far from the optimum, when communications are restricted.

Population $\mathscr{P}$	Resource Set $\mathscr{R}$		
	r_1	r_2	r_3
0	3	1	9
1	1	4	1
2	10	2	3

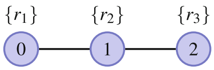

Fig. 1 Example of agents preferences and social graph

Population $\mathscr{P}$	Resource Set $\mathscr{R}$		
	r_1	r_2	r_3
0	2	10	4
1	5	3	9
2	2	7	1

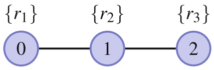

Fig. 2 Example of agents preferences and social graph

Proposition 1 (Social graph impact) *Independently of the objective function considered, a restricted social graph may prevent the achievement of optimal resource allocations.*

Proof Let us prove this proposition using a counter-example, based on a population of 3 agents $\mathscr{P} = \{0, 1, 2\}$ and a set of 3 resources $\mathscr{R} = \{r_1, r_2, r_3\}$. The agents preferences and the social graph with the initial resource allocation are described in Fig. 1. The objective is the maximization of the utilitarian welfare.

Agents only perform transactions increasing their own utility. According to such conditions and to the social graph, no transaction can be performed. Only two exchanges are possible, but both lead to a decrease of the individual welfare of at least one participant. The exchange of r_1 and r_2, or the exchange of r_2 and r_3 penalizes both participants.

However, the current allocation is suboptimal. The exchange of r_1 and r_3, which leads to an increase of both participants' utility, is not possible since agent 0 and agent 2 cannot communicate. Hence, restrictions on agents communication abilities may prevent the achievement of optimal solutions.

Restricted social graphs also have an indirect influence on the negotiation process. The order according to which agents negotiate is not important when complete social graphs are considered. Indeed, resources can always be traded with all other agents. However, this order becomes an important parameter to consider in the case of a restricted social graph.

Proposition 2 (Negotiation order) *Independently of the objective function which is considered, the order in which agents negotiate with each other may prevent the achievement of optimal resource allocations.*

Proof Let us prove this proposition using a counter-example, based on a population of 3 agents $\mathscr{P} = \{0, 1, 2\}$ and a set of 3 resources $\mathscr{R} = \{r_1, r_2, r_3\}$. The agents preferences and the social graph with the initial resource allocation are described in Fig. 2. The objective is the maximization of the utilitarian welfare.

Let us assume that agent 1 initiates a negotiation. Depending on which neighbor it selects to negotiate first, the negotiation process may end with sub-optimal allocations instead of optimal ones. If agent 1 first chooses agent 0 as partner, the exchange leads to a sub-optimal allocation from which the negotiation process cannot leave. However, if agent 2 is selected first, the negotiation process ends on a socially optimal allocation. Hence, the optimum can only be achieved using a specific order of negotiation.

Thus, the social graph represents an important issue since its topology may prevent the achievement of an optimal resource allocation in practice. Its influence on the efficiency of negotiation processes must be considered and should not be omitted as it has been done in former studies.

3 Solving Approach Characteristics

3.1 Resource Nature

The nature of considered resources (Chevaleyre et al. 2006) deeply impacts the properties of allocations, and then the way to negotiate resources efficiently. That is why it is important to clearly characterized the problems addressed. We choose to consider unique and atomic resources which are not shareable. Agents cannot alter the resources they own, they are only able to transact them. A resource allocation can be defined as follows:

Definition 6 (*Resource allocation*) Given a set $\mathscr{R}$ of m resources and a population $\mathscr{P}$ of n agents, a resource allocation A is represented as an ordered list of n resource bundles $\mathscr{R}_i \subseteq \mathscr{R}$ describing the subset of resources owned by each agent i:

$$A = [\mathscr{R}_1, \ldots, \mathscr{R}_n], \quad 1, \ldots, n \in \mathscr{P}, \quad A \in \mathscr{A}.$$

where $\mathscr{A}$ is the set of all possible allocations. The ith element of an allocation A corresponds to the resource bundle of agent i.

3.2 Agent Preferences

Agents express preferences over the resource set, which are used to determine their individual welfare (Doyle 2004). We choose to use a cardinal quantitative representation: an additive utility function.

Definition 7 (*Utility function*) An agent evaluates its individual welfare thanks to an additive utility function $u_i : 2^{\mathscr{R}} \rightarrow \mathbb{R}$. When agent $i \in \mathscr{P}$ owns a set of resources $\rho \subseteq \mathscr{R}$, its utility is evaluated as follows:

$$u_i(\rho) = \sum_{r \in \rho} u_i(r), \quad i \in \mathscr{P}, \quad \rho \subseteq \mathscr{R}.$$

Many other forms of agent preferences exist: the k-additive form which can express complementarity between resources for instance, the X- OR form mostly used in combinatorial auction or the *weighted propositional* form which make an explicit use of logic to express all kinds of synergy between resources. However, these forms are often used in a very specific domain whereas additive functions are the most widely used function in spite of the implicit restrictions.

3.3 Social Graph

Relationships between agents are modeled using a social graph. The topology of such a graph defines “who knows who” in the population. The social graph is a virtual entity: no agent is aware of the whole graph. It is indeed distributed between agents thanks to the notion of neighborhood. According to our realistic assumptions, agents have restricted perceptions of their environment.

Definition 8 (*Social graph*) The social graph $\mathscr{G}$ is a graph of relationships describing the communication possibilities among agents of a population $\mathscr{P}$. In such a graph, nodes represent agents, and an edge between two nodes means that the corresponding agents are able to communicate.

The different classes of social graphs can be grouped into three main classes:

– Complete graphs;
– Structured graphs;
– Random graphs.

First, negotiation processes based on complete social graphs can be compared to centralized approaches. Indeed, both of them assume complete communication abilities for all agents, and then have similar solving conditions.

Then, graphs with regular topological characteristics belong to the class of structured graphs. For instance, a graph where all agents have the same number of neighbors belongs to this class. Structured graphs also include some specific topologies like grids, rings or trees.

Finally, graphs with an irregular topology belong to the class of random graphs. Several classes exist (Bollobás 2001), like Erdős–Rényi graphs (Erdős and Rényi 1959), free-scale graphs or small worlds generated either by preferential attachment or by circle rewiring (Albert and Barabási 2002). The probability distribution is uniform when Erdős–Rényigraphs are considered, whereas the probability depends on the neighborhood size in small-worlds.

These graphs correspond to a representative sample of what can be encountered in most applications. Indeed, their characteristics vary significantly when these graphs are evaluated with the most widely used metrics (Biggs et al. 1986), like the mean connectivity, the clustering coefficient or the mean-shortest path length.

3.4 Transaction

During negotiation processes, the resource allocation evolves step by step by means of local transactions among agents. The resource traffic is generated thanks to these transactions, which move resources successively from an agent’s bundle to another one. Only bilateral transactions, i.e. transactions involving two agents simultaneously, will be considered in this paper. A transaction can be viewed as the association of participants’ offer. We choose to represent transactions in a parametric way: a transaction is characterized by the number of resources participants can propose.

Definition 9 (*Bilateral transactions*) A bilateral transaction between two agents i, $j \in \mathscr{P}$, denoted by δ_i^j, is initiated by agent i who involves a partner j. It is a pair $\delta_i^j \langle u, v \rangle = (\rho_i^\delta, \rho_j^\delta)$, where the initiator i offers a set ρ_i^δ of u resources ($\rho_i^\delta \subseteq \mathscr{R}_i$) and its partner j offers a set ρ_j^δ of v resources ($\rho_j^\delta \subseteq \mathscr{R}_j$).

This transaction representation can be used to model transactions from any class, like the ones described in (Sandholm 1998), using restrictions on the number of offered resources. During a *gift*, the initiator offers one resource and its partner provides nothing: it is then equivalent to a $\langle 1, 0 \rangle$-deal. In a *cluster*, the initiator may offer up to its whole resource bundle and gets nothing as counterpart. In a *swap*, both participants must provide a single resource, and finally, in a *cluster-swap*, both agents can offer a subset of resources.

3.5 Acceptability Criterion

The individual rationality is the most widely used criterion in the literature. It specifies that agents can only accept transactions increasing their individual welfare. It is used especially in the case of selfish agents.

Definition 10 (*Rational agent*) A rational agent only accepts a transaction that increases its own utility value. If the agent $i \in \mathscr{P}$ is rational, an acceptable transaction must satisfy the following condition:

$$u_i(\mathscr{R}_j) > u_i(\mathscr{R}_i), \quad i \in \mathscr{P}, \quad \mathscr{R}_i, \mathscr{R}_j \subseteq \mathscr{R}.$$

With respect to the social criterion, the welfare of the whole society cannot decrease. Sociability is more flexible than rationality, but experiments have shown its inefficiency (Nongaillard and Mathieu 2009a). Social agents are usually qualified as generous.

Definition 11 (*Social agent*) A social agent is an agent who only accepts transactions that increase the welfare of the whole society.

$$sw(A') > sw(A), \quad A, A' \in \mathscr{A}.$$

The social criterion is centered on the social welfare value, which is a global notion. Its value can only be determined thanks to the welfare of all agents. Agents should then know the resource bundle and the preferences of all agents in the population, in order to determine the value associated with the objective function. Such conditions cannot be satisfied since agents have only local information. The social value of the objective cannot then be locally computed. But, the computation of the exact value of the welfare function is not essential, to know its evolution is sufficient to determine whether or not a transaction penalize the society. Such computations can be restricted to the local environment of agents. If participants to a negotiation consider the remaining population as a constant, the evolution of the social value can be determined on a local basis. The formal definition of social transactions can be specified according to the welfare notion. The expressions of the conditions that transactions must satisfy, once applied to a specific social welfare notion, are detailed in Sect. 5.

3.6 Agent Behavior

Behaviors define agents from an external point of view. They describe how agents interact with each other, i.e. how they negotiate. During a negotiation, each agent makes and receives offers, and check their acceptability according to its own criterion. If a transaction is acceptable for every participant, it is performed. Otherwise, agents have to decide who has to modify its offer according to their behavior, and thus the negotiation continues.

Let us assume that agent $i \in \mathscr{P}$ initiates a negotiation and proposes an offer to of its partner $j \in \mathscr{N}_i$ previously selected. Both offers correspond to a bilateral transaction δ_i^j. If both agents consider this transaction acceptable, it is performed. However, if one participant rejects the offer, three alternatives can then be considered:

- agent i gives up and ends the negotiation;
- agent i changes the selected partner;
- agent i changes its offer or asks to change its partner's offer.

Determining the order of these actions is an important issue. Many behaviors have been implemented and tested, but only the most efficient one is presented here. The initiator always sorts the list of possible subsets it can offer according to its preferences. The initiator can then offer the least penalizing subset first. The initiator $i \in \mathscr{P}$ can change partners as well as its offer during a negotiation process. The initiator i proposes each offer $\rho_i \in L_i(\rho)$ to all its neighbors $j \in \mathscr{N}_i$ before changing it. Such an agent behavior is called *frivolous flexible* and is described in Algorithm 1.

Algorithm 1: Frivolous and flexible agent behavior

Input: Initiator i
Output: TRUE if a transaction is performed

```
L_i(ρ) ← generate(𝒯, ℛ_i) ;  // list of all possible generated offers
Sort L_i(ρ) according to u_i ;
Shuffle 𝒩_i ;
forall the ρ ∈ L_i(ρ) do                          // flexibility
    forall the j ∈ 𝒩_i do                         // frivolity
        forall the ρ' ∈ L_j(ρ) do
            δ ← (ρ, ρ') ;
            if TEST then                          // acceptability test
                Perform δ ;
                End the negotiation ;
                return TRUE ;
return FALSE ;
```

According to such a behavior, if an acceptable transaction exists somewhere in the neighborhood, it will necessarily be identified. The neighborhood should be shuffled between two negotiations in order to modify the order in which neighbors are considered, and thus avoid a bias.

4 Simulations and Protocol

Simulations are characterized by the number of agents and by the mean number of resources per agent. In this paper, 50 agents are negotiating 250 resources according to different settings. Agents can be either *rational* or *social*. Agents negotiate according to a negotiation policy, which is characterized by the size of agents' offers: $\langle 1, 1\rangle$ means that agents can only perform swaps whereas "up to $\langle 2, 2\rangle$" means that agents can propose up to two resources. It can also be explicitly written as: $\mathscr{T} = \{\langle 1, 0\rangle, \langle 0, 1\rangle, \langle 1, 1\rangle, \langle 2, 1\rangle, \langle 1, 2\rangle, \langle 2, 2\rangle\}$. Simulations are performed on social graphs that belong to different classes: complete, grids, Erdős–Rényiand small worlds. Random graphs are generated using a probability p that affects their characteristics. In this study, the link probability p varies from 0.05 up to 1.0. Each simulation is iterated 100 times from different initial resource allocations randomly generated, in order to evaluate the topological sensitivity.

5 Bilateral Negotiations

This section is dedicated to the evaluation of negotiation processes according to four social welfare notions. For each of them, the efficiency is first evaluated, by a comparison with the optimal social value, as well as the topological sensitivity. Then, the impact of the graph connectivity is evaluated.

5.1 Utilitarian Case

Table 1 presents the efficiency of negotiation processes based on different negotiation policy and on different classes of social graphs. This table shows the proportion of the optimal welfare value that can be achieved (left-side of the cells). The greater is the proportion, the closer optima are the resulting allocations. The deviation shows the proportion according to which may vary the provided solution. A large standard deviation means a high topological sensitivity. For instance, negotiation processes based on a grid where rational agents negotiate using $\delta\langle 1, 1\rangle$ transactions only end on social values representing 79.0 % of the optimum with a standard deviation of 1.6 %. Depending on the initial resource allocation, the utilitarian welfare value achieved may vary of 1.6 %.

Table 1 Utilitarian efficiency (%) and its deviation (%) according to the class of social graphs

Social graph class	Rational policy				Social policy							
	⟨1, 1⟩		Up to ⟨2, 2⟩		⟨1, 0⟩		⟨1, 1⟩		Up to ⟨1, 1⟩		Up to ⟨2, 2⟩	
Full	96.6	0.3	97.0	0.2	100	0	98.3	0.2	100	0	100	0
Grid	79.0	1.6	81.3	1.3	86.2	0.9	85.3	1.1	86.1	0.9	86.1	0.9
Erdős–Rényi	94.8	0.5	95.0	0.4	98.9	0.1	97.1	0.2	98.9	0.1	98.9	0.1
Small world	80.8	2.0	84.8	1.3	91.4	0.8	90.0	1.0	90.2	0.8	90.3	0.8

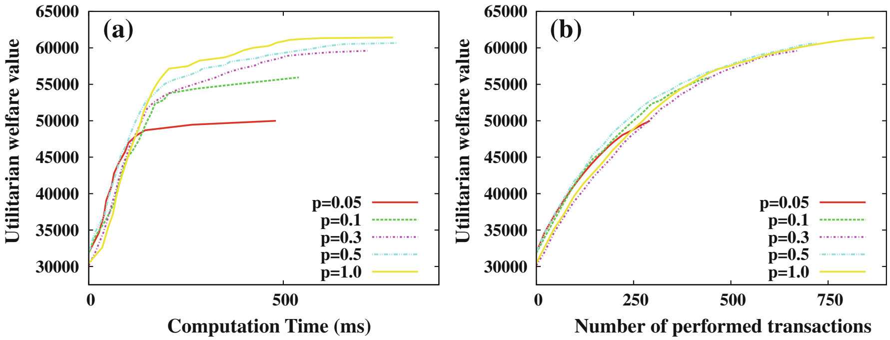

Fig. 3 Mean connectivity impact in terms of computation time **a** and of performed transactions **b**

Independently of the social graph class, rational negotiation processes always lead to socially weaker allocations than social negotiation processes. The restrictive character of the acceptability criterion affects the resource circulation, and then the quality of the provided solution. When considering complete social graphs, different negotiation policy always lead to optimal resource allocations. However, the use of large offers leads to important additional costs.

Negotiation processes lead to allocations associated with up to 98.9 % of the optimal welfare value when Erdős–Rényigraphs are considered. Only 91.4 % of the optimum is achieved when small-worlds are considered. In an Erdős–Rényigraph, the probability for a link to exist between any pair of nodes is constant, while in small-worlds, the larger is the number of neighbors, the higher is the probability to link this agent. Many agents have only one neighbors, and the resource traffic is unequally distributed. Then, bottlenecks, i.e., agents who block the resource circulation, may appear. When grids are considered, social negotiation processes achieve up to 86.2 % of the optimum. A weak mean connectivity handicaps the resource traffic and hence the achievement of socially efficient allocations.

The more restricted are social graphs, the weaker is the negotiation efficiency, and the higher is the deviation. In all cases, the standard deviation observed among the social values achieved remains small. It means that when the utilitarian welfare is considered, the topology has not a significant impact for a given class. The deviation is higher with rational negotiations since they restrict more the resource traffic, which then influences on the solution quality. The more restricted is the resource traffic, the higher is the standard deviation, and thus more important become the initial resource allocation.

The social graph topology greatly affects the resource circulation and the negotiation efficiency. The larger are agent neighborhoods, the denser are social graphs, and the easier is the resource traffic. The probability p for a link to exist between nodes from any pair can be modified.

Figure 3 shows the impact of the social graph connectivity on the efficiency within a population negotiating using social gifts. Figure 3a represents the evolution of the utilitarian welfare value according to the computation time, whereas Fig. 3b represents its evolution according to the number of performed transactions.

Table 2 Egalitarian efficiency (%) and its deviation (%) according to the class of social graphs

Social graph class	Rational policy				Social policy							
	⟨1, 1⟩		Up to ⟨2, 2⟩		⟨1, 0⟩		⟨1, 1⟩		Up to ⟨1, 1⟩		Up to ⟨2, 2⟩	
Full	19.3	62.9	20.8	73.9	78.5	1.8	24.1	28.7	99.9	0.3	99.9	0.3
Grid	13.9	71.3	14.6	80.2	66.2	4.1	23.6	29.6	80.2	1.8	80.6	1.7
Erdős–Rényi	17.4	71.9	20.2	76.8	77.3	2.2	23.8	27.3	96.1	6.8	96.6	6.5
Small world	13.1	73.0	13.9	77.5	63.8	10.4	23.4	27.8	78.1	9.4	78.2	10.5

These figures show that a weak probability, i.e. small neighborhoods, leads to short transaction sequences and a welfare value far from the optimum. For instance, when $p = 0.05$, negotiations end after a sequence of 300 gifts performed in only 0.5 s. However, the efficiency of negotiation processes decreases by 20 %. The gradual increase of the probability p leads to longer transaction sequences, to the achievement of larger utilitarian values, and to more time-consuming negotiations. Larger neighborhoods facilitate the resource circulation by offering a larger number of possible transactions to all agents. The impact becomes really significant when $p < 0.3$. Above this value, the resource circulation is sufficient to achieve socially interesting allocations, but below this threshold, social graphs are too restricted, and the flexibility of the social criterion cannot compensate for the restrictiveness of graph topologies.

5.2 Egalitarian Case

Table 2 shows the impact of the social graph topology on the egalitarian negotiation efficiency and the associated deviation.

Table 2 shows that, generally, negotiations among rational agents achieve unfair allocations. Indeed, independently of the allowed transactions, independently of the social graph topology, rational negotiations end quite far from the optimal welfare value. Only 20 % of the optimal welfare value is achieved in the best cases. The standard deviation of negotiations among rational agents is very important. In the case of rational negotiations based on small-worlds, egalitarian welfare values that can be achieved may vary by 73 %. Initial resource allocations and graph topologies are the most important factor when rational egalitarian negotiations are considered. Thus, the rationality criterion is definitively not well-adapted to solve egalitarian problems efficiently. It restricts the set of possible transactions too much and throws negotiation processes into local optima. Generosity is hence an essential feature in order to achieve fair allocations.

Even using on complete graphs, no social negotiation policy can guarantee the achievement of egalitarian optima. Whereas social gifts are well adapted to the solution of utilitarian problems, they do not suit to the case of egalitarian problems. Only 78.5 % of the optimum can be achieved in the best cases. Indeed, after a finite number of transactions, agents can not give any additional resource without becoming poorer than their partners. Negotiations based on social swaps lead to severely sub-optimal

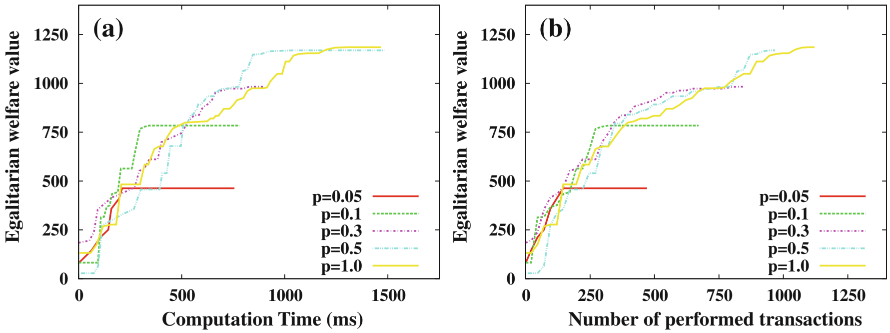

Fig. 4 Influence of the mean connectivity on egalitarian negotiations in terms of the computation time in (**a**) and of number of performed transactions in (**b**)

resource allocations with an efficiency of 24.1 % on complete social graphs in the best case. Such a weak efficiency is mainly due to the inherent constraints of swap transactions. Since the resource distribution cannot be modified, a poor agent who has only few resources initially, penalizes a lot the egalitarian negotiation process. When both gifts and swaps are allowed, the negotiation efficiency is really close to the optimum. Larger bilateral transactions improve only a little the fairness among agents, but are much more expensive to determine.

Social graphs of weaker mean connectivity like grids lead negotiation processes to socially weaker allocations. When small-worlds are considered, the resource traffic is restricted by the large number of agent leaves, leading to a larger deviation. Indeed, if such agents cannot identify an acceptable transaction with their lone neighbor, some resources may be trapped in the bundle of such agents.

The social graph topology influences a lot the resource circulation as well as the efficiency of negotiation processes. The larger are agents' neighborhoods, the denser are social graphs, and consequently resources can circulate easily. The probability of link generation between two agents can be modified to evaluate its impact. High probabilities correspond to dense social graphs.

Similarly to utilitarian negotiations, Fig. 4 show that a high probability, which corresponds to a dense social graph, leads to longer sequences of transactions during negotiation processes, which achieve moreover a higher welfare value. Larger neighborhoods facilitate the resource circulation by offering larger numbers of possible transactions to all agents. The impact of the connectivity is important only if the probability p of link generation is very low. The impact of the connectivity is not linear, it becomes really significant below $p \leq 0.3$.

5.3 Nash Case

When the Nash welfare is considered, the efficiency of the negotiation processes can be evaluated using a comparison with estimation provided by heuristic, as described e.g. Nongaillard et al. (2009). Table 3 shows the efficiency of negotiations depending

Table 3 Nash efficiency (%) and its deviation (%) according to the class of social graphs

Social graph class	Rational policy				Social policy							
	⟨1, 1⟩		Up to ⟨2, 2⟩		⟨1, 0⟩		⟨1, 1⟩		Up to ⟨1, 1⟩		Up to ⟨2, 2⟩	
Full	99.9	0.33	100.1	0.27	101.6	0.06	100.1	0.31	101.7	0.02	101.7	0.02
Grid	97.0	0.44	97.5	0.40	99.6	0.14	98.2	0.37	99.7	0.14	99.7	0.14
Erdős–Rényi	99.6	0.33	99.8	0.28	101.4	0.06	99.9	0.32	101.6	0.02	101.6	0.02
Small world	97.2	0.46	98.0	0.38	100.2	0.13	98.9	0.37	100.4	0.12	100.4	0.12

on the class of social graphs considered, and the standard deviation which correspond to the topological sensitivity.

Table 3 shows that some welfare values achieved >100 %. Since heuristics can only give an estimation of Nash welfare values, an efficiency >100 % means that the corresponding negotiation processes lead to socially more interesting allocations than the ones provided by the heuristics. Moreover, the difference may seem small between values but it is a consequence of the use of the Logarithm function which is essential to avoid too large numbers.

Rational negotiations generally achieve socially weaker allocations than social negotiations. Allowing gifts and swaps during a negotiation process seems sufficient to achieve socially efficient allocations. Larger transactions do not significantly improve the Nash welfare values achieved while the negotiation cost that increases a lot. Similarly to the egalitarian case, negotiations based on swap transactions achieve the socially weakest allocations. Since the initial resource distribution cannot be modified, negotiations end quickly on local optima. The deviation is also higher than for other transactions. Negotiation processes based on grids leads to the weakest allocations. The mean connectivity of the social graphs is an important feature, deeply affecting the negotiation efficiency. Relationships among agents are too restricted to allow a suitable resource traffic, and then prevent the achievement of optimal allocations. The comparison between results achieved on Erdős–Rényigraphs and the ones achieved on small-worlds indicates that a large number of agent-leaves penalizes a lot the negotiation process.

The social graph topology affects the negotiation efficiency and may prevent the achievement of socially optimal allocations. Agent relationships are represented here by Erdős–Rényigraphs, which are generated with different probabilities. The variation of the probability influences on the number of links, and thus the mean neighborhood size.

Figure 5a represents the Nash welfare evolution according to the elapsed time, and Fig. 5b corresponds to the Nash welfare evolution according to the number of performed transactions. They show that denser social graphs lead to longer negotiation processes (with larger number of performed transactions) and to a higher utilitarian welfare value at the end of the negotiation process. The connectivity has an important influence only if the probability p of link generation is very low. The influence of the connectivity is not linear, it becomes really significant below $p \leq 0.3$.

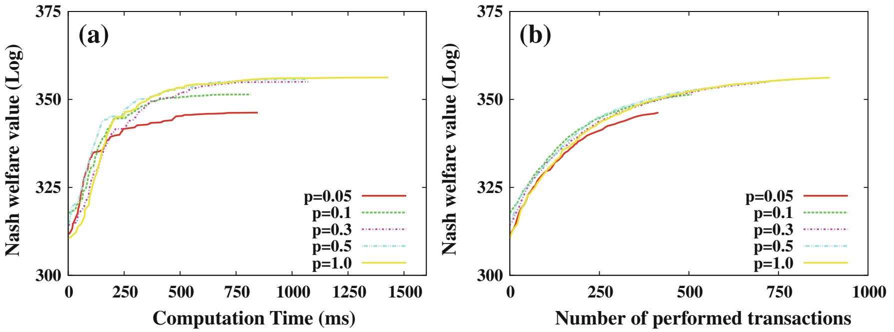

Fig. 5 Influence of the mean connectivity on Nash negotiations according to the computation time in (**a**) and to the number of performed transactions in (**b**)

Table 4 Elitist negotiation efficiency (%) and its standard deviation according to the class of social graphs

Social graph	Social efficiency (%) $\langle m_i, 0\rangle$	Standard deviation
Full	100	0
Grid	31.17	26.92
Erdős–Rényi	95.12	11.53
Small world	68.43	66.50

5.4 Elitist Case

According to the elitist acceptability criterion, restrictions can be made on the cardinality of the allowed transactions. Since all utility values are positive, an implicit consequence can be observed: An elitist negotiation process tends to gather all resources into a single agent bundle, who assigns the largest utility value to them. Then, any bilateral transaction $\delta_i^j\langle u, v\rangle$ such that $v > 0$ is contrary to this objective. Several sizes of offers can be tested. However, the computation time required to end an elitist negotiation processes based on $\delta_i^j\langle 1, 0\rangle$ for instance is exponentially higher $(i, j \in \mathscr{P})$. Thus, only cluster transactions of maximal size, i.e., $\langle \rho_i^\delta, 0\rangle$ transactions should be used when negotiating.

Since only cluster transactions are considered, an acceptability criterion cannot be used anymore. Indeed, since agents only offer their whole resource bundle without compensation, the rational acceptability criterion is improper. Since all utility values are positive, no rational cluster transaction exists. Thus, such an acceptability criterion is unadapted and inefficient when the elitist welfare notion is considered. This acceptability criterion is thus not represented in the following experiments.

Table 4 presents the efficiency of elitist negotiation processes based on social $\delta\langle m_i, 0\rangle$ transactions and on different classes of social graphs. Here, 50 agents negotiate 250 resources.

Table 4 shows that, when the relationships among agents can be modeled by a complete social graph, negotiation processes based on cluster transactions of maximal size always lead to socially optimal resource allocations. When grids are considered,

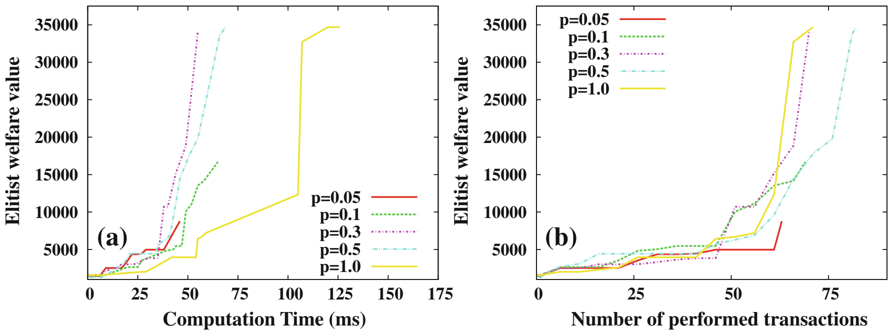

Fig. 6 Social graph connectivity impact on elitist negotiations in terms of computation time in (**a**) and of performed transactions in (**b**)

only 31.17 % of the optimum is achieved. The mean connectivity is too weak to ensure a proper resource traffic. Moreover, the quality of the solution may vary by 26.92 %. The large standard deviation reveals the important impact of the topology. Since resources circulate barely, depending on the agent to which resources are initially allocated, resources can be trapped somewhere, and then penalize elitist negotiations. Resources remain dispatched over the population, which explains the poor efficiency of negotiations based on grids. Negotiations based on Erdős–Rényigraphs achieve socially efficient allocations. Indeed, 95.12 % of the optimal welfare value can be achieved, with a standard deviation of 11.53 %. The mean connectivity is higher than in grids, which allows a sufficient resource circulation and result in interesting allocation. However, in the case of small-worlds, only 68.43 % of the optimal welfare value can be achieved. Their mean connectivity is really low (on average 6.8 neighbors per agent) and irregular. Most agents have only few neighbors while few agents have many neighbors. This irregularity explains the very large standard deviation which is observed. Depending on the initial resource allocations, many resources cannot change of owners and thus lead negotiation processes into a local optimum.

The mean connectivity of social graphs affects the negotiation efficiency since it restricts more or less the transaction possibilities. Considering Erdős–Rényigraphs, the mean connectivity can vary thanks to the probability p of link generation in the model of generation $G(n, p)$. These simulations are based on a population of 50 agents who negotiate 250 resources, carrying out frivolous and flexible behaviors.

As shown in Fig. 6b, the number of performed transactions does not vary significantly (between 65 and 80). Negotiation processes end after transaction sequences of close length. Figure 6a shows on the other hand that the computation time varies from 40 to 125 ms. However the elitist welfare value on which negotiation processes end vary a lot. The mean connectivity significantly affects the quality of provided solutions only when the probability is below $p = 0.3$. If the probability of generating a link between two nodes is higher, the efficiency is not affected more than 8 %. But if the probability is lower, the elitist welfare value that can be achieved decreases drastically.

6 Conclusion

Many studies focused on allocation problems and solutions are proposed, often through centralized techniques. However, if resources are initially distributed between agents within the population, it becomes a reallocation problem. Thus, the aim is to find a transaction sequence leading from the initial allocation to an optimal solution. Whereas centralized techniques are not adapted due to the problem complexity, former agent-based approaches were based on a ideal context. Indeed, agents were omniscient and were able to negotiate with all agents of the population. However, such assumptions are not realistic compared to real life applications.

In this paper, we propose an approach based on a more realistic context, where agents have limited perception and limited knowledge. They starts negotiating knowing only their resource bundle, their own preferences and a list of possible partners. Any kind of communication restrictions can be modeled using the method we propose. We show that it is important to consider the social graph in order to guarantee the negotiation efficiency in real conditions. We identify the characteristics favoring and penalizing the negotiation efficiency according to different negotiation settings and different welfare notions.

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