Efficient algorithms for the design of finite impulse response digital filters

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Digital filters represent the foundation for all that is digital signal processing, with widespread applications ranging from data transmission to audio and image processing. A filtering toolchain is comprised of three major steps :

— **derive a concrete mathematical representation for the filter in terms of polynomials or rational functions**;

— quantization of the filter coefficients using fixed-point or floating-point numerical formats;

— hardware synthesis of the filter.

We will be concerned with the first step. One of the best known routines for designing digital filters is the Parks-McClellan [4] algorithm. It is an extension of the well known Remez [1] algorithm for minimax polynomial approximation of functions. The problem it tries to solve can be stated in terms of approximating a continuous function on a union of closed intervals over the reals by means of a linear combination of Chebyshev polynomials.

One of the reasons this routine has enjoyed such a wide adoption in the signal processing community is its practical robustness. In this talk we will describe a new implementation of this iterative algorithm which uses recent results related to barycentric Lagrange interpolation [5] and a numerically stable root finding routine based on determining the eigenvalues of appropriate generalized companion matrices of polynomials [6]. To this end, it shares the same design philosophy as the Remez routine available inside the Chebfun package [2, 3]. To justify the benefits of using our implementation, we will compare it to the de facto one available in Matlab.

We will also try to give some theoretical arguments as to why this filter design routine behaves well in practice, by looking at the numerical stability of the formulas we are using. In particular, our analysis is based on the fact that barycentric Lagrange interpolation is backward stable when a certain Lebesgue constant is small [7].

Bibliography

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