Symbolic Computation for Boundary Problems and Green's Operators

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Classical differential algebra provides powerful methods for analyzing and manipulating differential equations (both linear and nonlinear) but lacks tools for dealing with boundary conditions. In [1, 2] we have introduced a new paradigm for incorporating integral operators (also known as Rota-Baxter operators) into ordinary differential algebras. This allows us to formulate, compute and factor the solution operator (usually called Green's operator) of two-point as well as Stieltjes boundary problems for a linear ordinary differential equation (LODE), relative to a given fundamental system.

In the case of linear partial differential equations (LPDEs), the abstract algebraic framework applies [3], but the formulation of a suitable operator ring over a given partial differential algebra is more subtle (this is work in progress).

We shall give a short survey of the LODE case and some hints about the innovations needed for dealing with LPDEs. A short application in the area of actuarial mathematics illustrates the power of the factorization approach for boundary problems [4, 5]. We round up the talk with a short demo of Anja Korporal's Maple package `IntDiffOp` [6].

Bibliographie

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