Computing the Rank Profile Matrix

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The row (resp. column) rank profile of a matrix describes the stair case shape of its row (resp. column) echelon form. We will first propose a recursive Gaussian elimination algorithm that can compute simultaneously the row and column rank profiles of a matrix, as well as those of all of its leading sub-matrices, in the same time as state of the art Gaussian elimination algorithms. We then propose the definition of a new matrix invariant, the rank profile matrix, summarizing all information on the row and column rank profiles of all the leading sub-matrices. We also explore the conditions for a Gaussian elimination algorithm to compute all or part of this invariant, through the corresponding PLUQ decomposition. As a consequence, we show that the classical iterative CUP decomposition algorithm can actually be adapted to compute the rank profile matrix. Used, in a Crout variant, as a base-case to the recursive algorithm, it delivers a significant improvement in efficiency. The row (resp. column) echelon form of a matrix are usually computed via various dedicated triangular decompositions. We show here that, from some PLUQ decompositions, it is possible to recover the row and column echelon forms of a matrix and of any of its leading sub-matrices thanks to an elementary post-processing algorithm. Lastly, we will make connections with the recent algorithmic improvements of Storjohann and Yang [3, 4] for the computation of the rank profiles of matrices with small ranks, leading to an $O^{\sim}(r^{\omega} + mn)$ algorithm computing the rank profile matrix. This work has been presented, for the most part, in [1] and [2].

Bibliographie

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