sujet de stage de Master 2 recherche (2023)

Types for sensitivity analysis and differential privacy
in functional programming

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Duration: 4-6 months
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Context:
Program sensitivity bounds the distance between the outputs of a program when run on two related
inputs. This notion plays an important role in differential privacy, a rigorous approach for ensuring privacy
in database queries and data analysis computation. Several programming languages approaches to sensitivity
analysis and differential privacy have been developed in the last decade [BGHP16]. Among them are type
systems inspired by linear logic, as introduced in the Fuzz programming language [RP10, GHH+13]. In
Fuzz, each type is equipped with its own notion of distance, and sensitivity analysis is carried out by type
checking. The language is also equipped with a monadic type for probabilistic computation. This leads to
theorems stating that if a program is well-typed in this system, then it is differentially private.

Fuzz was designed to account for two notions of distances on product types: \( L_1 \) (or Manhattan) distance
and \( L_\infty \) (or Chebyshev) distance. This is because these distances play an important role in differential
privacy. However, more general \( L_p \) distances (such as e.g. euclidean distance \( L_2 \)) are used in optimisation,
information theory and statistics. In [jwdABG22] (joint work with colleagues at Boston University) an
extension of Fuzz was proposed, called Bunched Fuzz, with a richer type system allowing to account for
arbitrary \( L_p \) distances. This system is inspired by the logic of Bunched Implications (BI) [OP99], hence its
name: instead of a set structure, typing environments have a bunch (tree) structure.

Objectives:
In this internship we propose to investigate and enlarge the study of properties of Bunched Fuzz. This
work could take one or several of the following directions, according to the interests of the candidate:

• Deepen the understanding of the meta-properties of Bunched Fuzz: one can for instance study a
subject-reduction property, for a certain operational semantics.

• Extension of typing rules for distances on probability distributions: Bunched Fuzz allows for the
definition of new distances on the type of probability distributions (such as Hellinger distance); one
can study which distances over distributions (defined by f-divergences) admit valid typing rules, which
subtyping rules can be added, and which probabilistic computation can be analysed thanks to these
new types.

• Comparison with other systems for differential privacy: The Duet language [NDA+19] can be seen as a
refinement of Fuzz allowing to analyse the more general notion of \((\epsilon, \delta)\)-differential privacy. However
it is not equipped with type constructs handling \( L_p \) distances. One can wonder if it could be refined in a
similar way as Bunched Fuzz. Another setting is that of the Fuzzi system [ZH+19], a 3-layered logic
(in the sense of program logics) providing a more fine-grained analysis of differential privacy. Could Fuzzi be extended to take into account $L_p$ distances?

**Expected background:** Some knowledge of type systems, functional programming or $\lambda$-calculus is needed. Some familiarity with linear logic or linear type systems would also be useful. Knowledge of differential privacy would be appreciated but is not compulsory.

**References**


