

# Implicit computational complexity in Pi-calculus

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## Context :

How can one define some simple and modular programming disciplines in a high-level programming or specification language in such a way that the corresponding programs exactly characterize a certain complexity class of functions, for instance polynomial time (FP) or polynomial space (FSPACE)? This is the goal of *implicit computational complexity* (ICC) which aims at providing such machine-independent characterizations for a large variety of complexity classes and of source languages, without referring to any explicit bound on time or space usage. This research area uses ideas and techniques coming from logic, recursion theory and type systems.

Most of ICC results have been obtained for sequential languages, initially for  $\lambda$ -calculus or the functional computing paradigm, and then for imperative and object-oriented languages. One of the classical results [BC92] is that of safe (or ramified) recursion which is a restriction of primitive recursion that characterizes the class FP of polynomial time functions. This approach is also illustrated by a type system for  $\lambda$ -calculus characterizing FP [LM93].

Just as  $\lambda$ -calculus represents sequential computation, process calculi such as  $\pi$ -calculus [SW03] have been introduced to represent parallel and concurrent computation. This language represents processes communicating by messages sent through channels. However up to now ICC characterizations in  $\pi$ -calculus have hardly been explored.

## Objective :

The goal of this internship is to give an ICC characterization of a complexity class in  $\pi$ -calculus. The paper [DY18] has defined a criterion on  $\pi$ -calculus processes inspired by safe recursion [BC92] and which ensures a polynomial bound on *causal complexity*. However it has not provided any expressivity result for the processes satisfying the criterion, for instance stating that say all FP or all FSPACE functions would be representable. On the other hand the paper [HMP13] has given a criterion on programs of an imperative language with threads which characterizes the class PSPACE (which is equal to parallel polynomial time, as shown by complexity theory). However this imperative language is not as general as the  $\pi$ -calculus.

More recently [BG21, BGK21, Ghy21, BG22] have provided type systems allowing to obtain bounds on the parallel time complexity of  $\pi$ -calculus processes. Note that these are *not* ICC characterizations because the bounds do not belong to a specific class (e.g. polynomial bounds).

We propose to look for a typing criterion in  $\pi$ -calculus in the style of safe recursion which would characterize a complexity class. The most natural candidate would be the class FSPACE. We suggest to use the techniques of [BG21, BGK21] to prove an upper bound on the complexity of the processes.

**Expected background :** It is expected to have some knowledge about  $\lambda$ -calculus or functional programming languages as well as about type systems. Some basic background on computational complexity is also necessary. Some notions of concurrent process calculi such as  $\pi$ -calculus or CCS would also be welcome but are not compulsory.

## Références

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