An LMS-based Grammar Self-Index with Local Consistency Properties

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Claude et al. 2020 demonstrated that there is a self-index on top of $\mathcal{G}$ that requires $O(g \lg n + (2 + \epsilon) g \lg r)$ bits of space, where $0 < \epsilon \leq 1$ is a constant. This index can locate the *occ* the occurrences of a pattern $P[1, m]$ in $O((m^2 + occ) \lg g)$ time.

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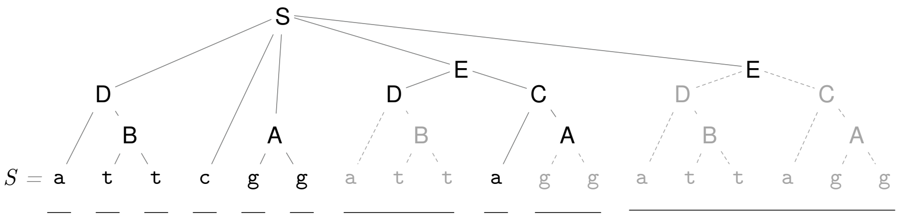

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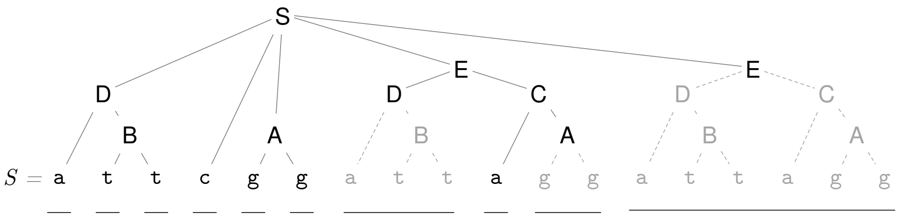

A $\rightarrow$ g g	g|g
C $\rightarrow$ a A	a|gg
S $\rightarrow$ D c A E E	att|c|gg|attagg|attagg
E $\rightarrow$ D C	att|agg
B $\rightarrow$ t t	t|t
D $\rightarrow$ a B	a|tt

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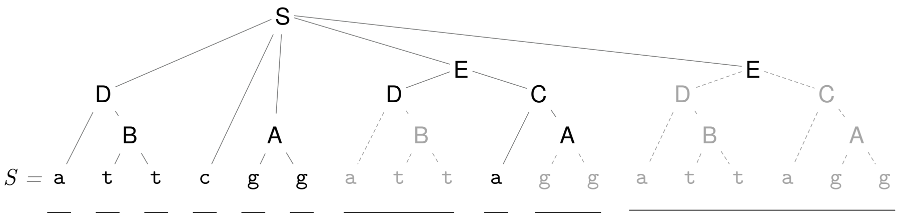

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C → a A	a|gg	$\mathcal{Y}$ $\mathcal{X}$
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E → D C	att|agg	c|ggattaggattagg
B → t t	t|t	gg|attaggattagg
D → a B	a|tt	attagg|attagg

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	agg	attagg	attaggattagg	cggattaggattagg	g	gg	ggattaggattagg	t	tt
a						C			D
c							S		
g					A				
gg			S						
agg									
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t								B	
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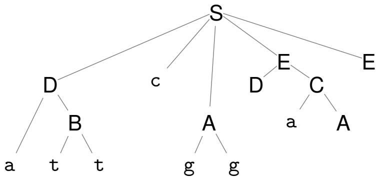

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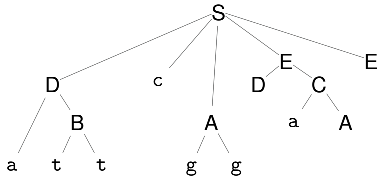

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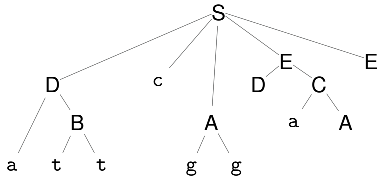

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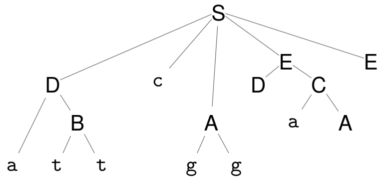

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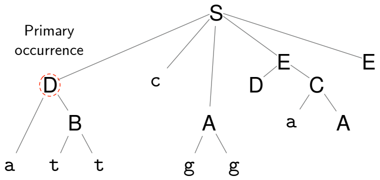

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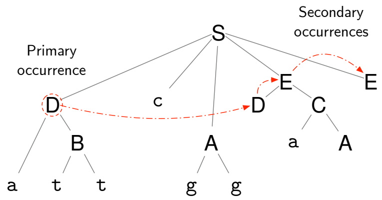

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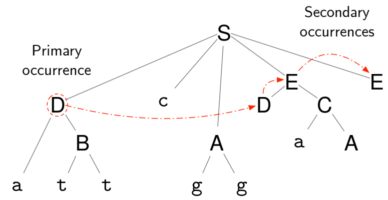

P = a | t t

Problem: We have to try out all the possible cuts of P

Locally consistent grammar self-index

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- It is balanced.
- The occurrences of a pattern P are largely compressed in the same way.

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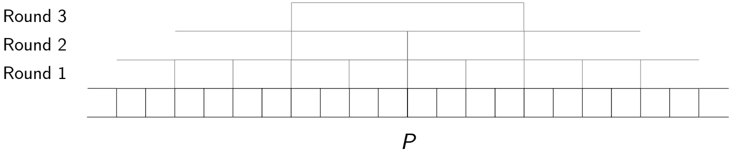

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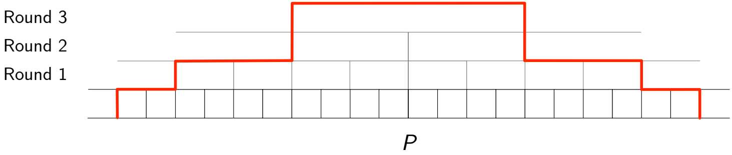

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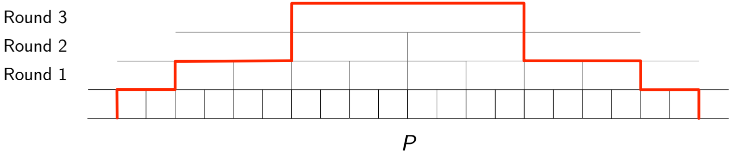

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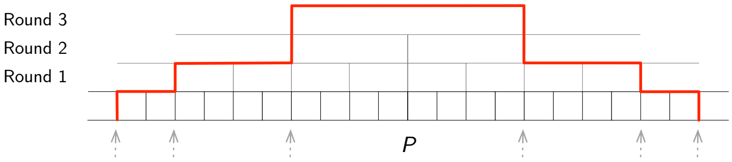

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- It increases the size of the grammar index considerably.

Our method

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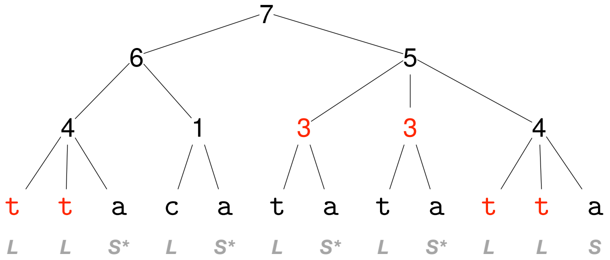

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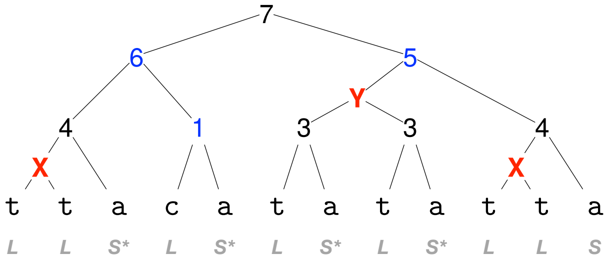

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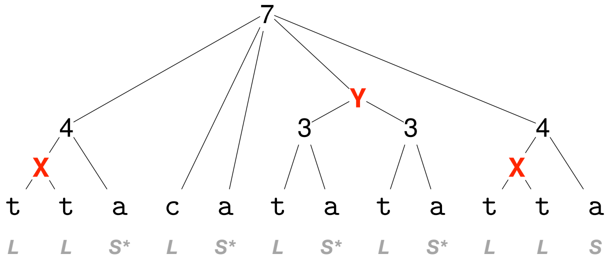

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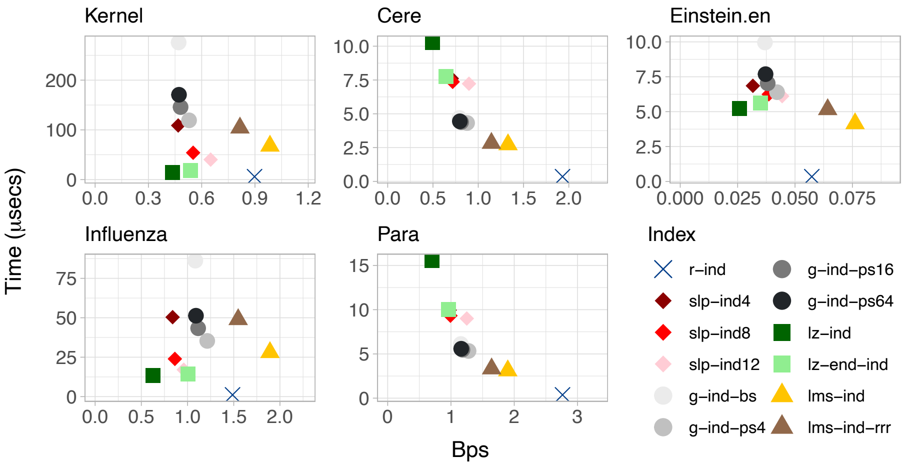
  - `r-ind`: the r-index.
- We assessed the space usage and the time for answering the *locate* operation.

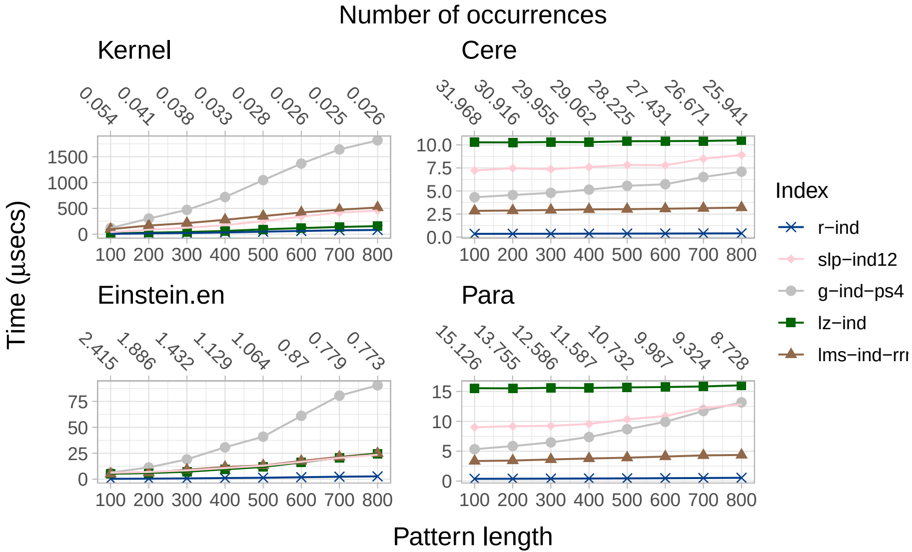
Results: grammar algorithm

Dataset	n	σ	RePair	LMS	LMS post	LC
para	429,265,758	5	5,344,480	22,787,047	8,933,303	8,888,002
cere	461,286,644	5	4,069,450	37,426,507	6,802,801	4,069,450
influenza	154,808,555	15	1,957,370	4,259,746	3,304,035	4,477,322
einstein.en	467,626,544	139	212,903	643,338	427,142	601,755
kernel	257,961,616	162	1,374,650	3,769,839	2,870,350	3,795,801

Results: self-indexes



Results: locate operation



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  - Is it possible to use the Wheeler graph framework to create a grammar-based self-index?
- Can we use the concept of locally consistent parsing to support inexact locate queries?
- What other types of queries can we support using local consistency?

Questions?