

A New Kind of Three-Dimensional Anamorphosis

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Abstract

We define a new kind of anamorphosis, where the distorted image is a three-dimensional object, whose reflection in a mirror restores a hidden design. A chain of software and programming languages are used to actually build the sculpture, using a 3D printer. This work, from the beginning of an idea, its elaboration through software tools and programs, plus the interaction between two parties involved, illustrates how fruitful a collaboration between artists and scientists can be.

1 Introduction

Catoptric anamorphoses are distorted images that need to be seen through a mirror, and from a specific point of view, to reconstruct the original image. Anamorphoses appeared in Europe at the time of Renaissance, when artists and scientists discovered the laws of perspective. A complete history of anamorphoses can be found in Jurgis Baltru aitis' *Anamorphoses ou Thaumaturgus opticus* [1].

The setting-up of an anamorphosis needs three items: a mirror, an observer, and the locus where the distorted image lies, called the *surface of distortion*. In a paper presented at Bridges 2010 [3], I described a general procedure for testing and constructing catoptric anamorphoses. This method computes the distorted image corresponding to an anamorphosis setting and outputs it, to obtain a physical anamorphosis. In [3], the surface of distortion was either a plane surface or a developable surface. Extensions of the method could also help create anamorphoses where the surfaces of distortion are no longer developable: we then lose the possibility of printing the distorted design, but are still able to create it by directly drawing on the surface, using the information returned by the program.

In October 2010, James Hopkins, a British sculptor [5], asked me whether it would be possible to compute the shape of a three-dimensional wired form sculpture which would represent a chair when seen through a spherical mirror.

This challenging question could have been asked in another way: "Is it possible to replace the *surface of distortion* with something like a *volume of distortion*?" The answer is "yes", as shown in figures 1 and 2. The rest of the paper will explain how we solved this problem. Section 2 will survey previous similar works. Section 3 recalls the method described in [3]. Section 4 explains how this general method was modified in order to generate three-dimensional distorted objects. Section 5 shows the application of the method with an example. Section 6 lists a non-exhaustive set of possible directions to explore and concludes this paper.

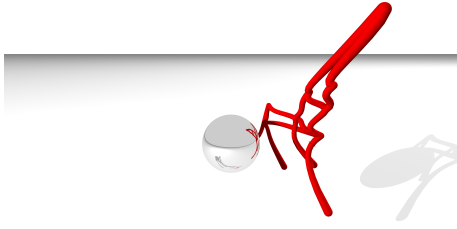


Figure 1 : *Reflected chair: the 3D sculpture.*

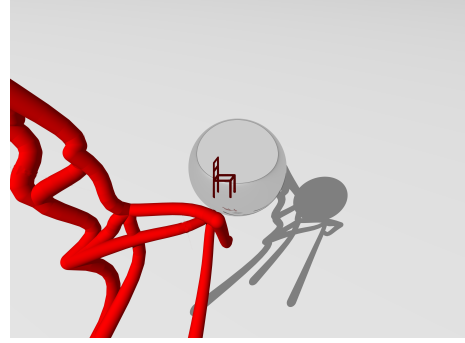


Figure 2 : *The sculpture seen through a reflective sphere.*

2 Previous Works

Among catoptric anamorphoses, those using cylindrical or conical mirrors are common and well studied ([6], [7]). Few studies have been done for other mirrors shapes, and very few concerning three-dimensional distorted sculptures (instead of distorted flat images): In 1984, Fujio Watanabe [8] exhibited two artworks, where a central conical mirror was surrounded by a plastic carved cylinder. The image of this cylinder, when viewed through the mirror, revealed a human face and a skull. Stella Battaglia and Gianni Miglietta [2] used spherical mirrors surrounded by three-dimensional sculptures. James Hopkins has recently made a sculpture where a distorted three-dimensional glass is reflected correctly in a cylindrical reflective bottle.

3 A General Method for the Construction of Catoptric Anamorphoses

Let us recall the general principle which gave rise to the method presented in [3]:

Figure 3 illustrates the principle behind the method. The designer of the anamorphosis wants the observer, placed at position V , to see a correct image in a mirror M . This image is obtained by reflection of a distorted image lying on a surface of distortion P . The image can be thought as lying on a virtual screen E . Let S_1 be a pixel from the image on E : when the observer looks at S_1 , ray R_1 , passing through V and S_1 , hits the mirror M at point T_1 . This ray is reflected and hits the plane at point W_1 . Reversing the direction of R_1 , we can say that W_1 is seen by the observer, who looks in the direction of the mirror, in the exact direction of S_1 . This reasoning is valid for each pixel. Then the distorted image is the union of all the W_i , each of them associated with a pixel S_i of the original image. Screen E , which was useful for the demonstration, can be removed from the installation. Pixel S_2 shows how the image can be distorted: two neighbouring pixels in the original image can result in very distant points on the surface of distortion.

In other words, computing a distorted image for an anamorphosis setting consists in following the light rays outgoing from the observer's eye, through the desired image, hitting the mirror and then the surface of distortion. Each such ray defines a pixel on the surface of distortion.

4 Adapting the General Method

Our problem is slightly different. If we want the distorted object to be a three-dimensional object, we must replace the notion of *surface of distortion* with something else. We decided to define a set of N surfaces of distortion $\{S_0, \dots, S_{N-1}\}$. The image to be distorted is divided into line segments (a 'wired' design). Each

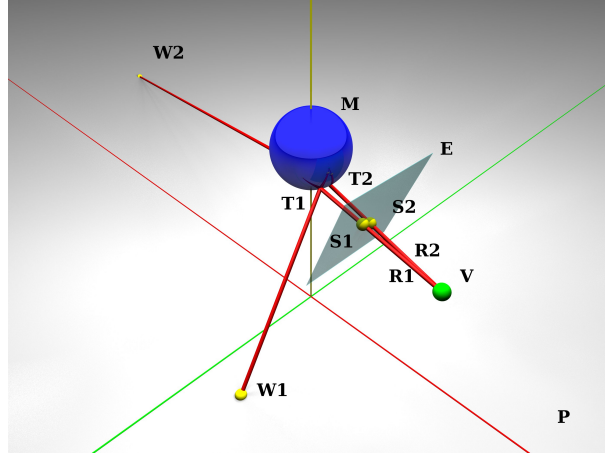


Figure 3 : *The Catoptric Anamorphosis Principle.*

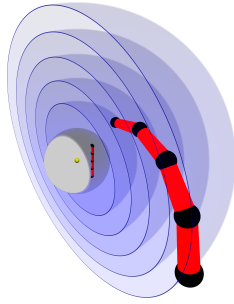


Figure 4 : *3D anamorphosis method: the line segment in the mirror is the reflection of the 3D curve. The yellow sphere in the center of the mirror is the observer's eye.*

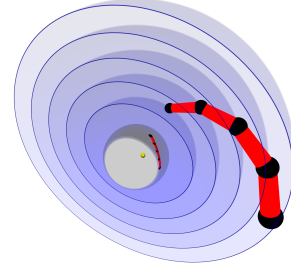


Figure 5 : *The same scene seen from another point of view.*

segment is itself divided into $N - 1$ sub-segments, leading to a set of N points $\{P_0, \dots, P_{N-1}\}$ corresponding to the endpoints of those $N - 1$ sub-segments. Using the method described in [3], we compute the image P'_i of point P_i on the surface of distortion S_i . When seen in the mirror, the curve connecting the P'_i in order is an approximation of the original segment. The approximation becomes better as N increases.

Figures 4 and 5 give an illustration of the process. Five concentric (virtual) spheres are wrapped around the spherical mirror: all together they define the *set of surfaces of distortion*. A vertical line segment is obtained by the reflection of a curve intersecting the five surfaces. Reflections of the black spheres are aligned. For a better result, one must use more than five surfaces (50 surfaces are typical). Also notice: the farther the cylinders are from the mirror, the thicker they must be, in order to be perceived with the same thickness when seen by reflection. Hence, the curve connecting the P'_i is not the union of cylinders, but more precisely a set of (ordered) truncated cones mitred with spheres to obtain a continuous shape. This poses no problem for the programs used to generate the anamorphosis.

A similar procedure is designed to define the distorted image of a circular arc, dividing it into $N - 1$ sub-arcs, and replacing each sub-arc with a truncated cone. Figures 6 and 7 illustrate this process. Hence, combining those two procedures, we can draw designs composed of line segments and circular arcs, and then

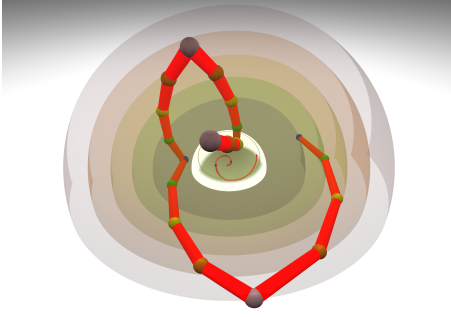


Figure 6: *Anamorphosis of the approximation of a logarithmic spiral, made with five circular arcs.*

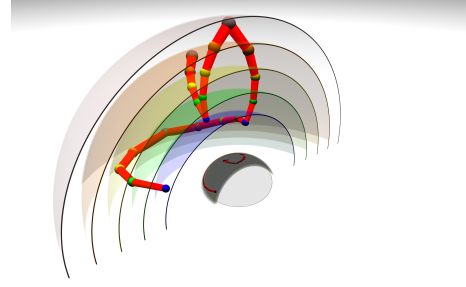


Figure 7: *The 3D sculpture seen from the side. Spheres indicate the intersection of the structure with the volume of distortion.*

compute their distorted three-dimensional image, given the position of the observer, the shape of the mirror and the set of surfaces of distortion.

As with the original method, all computations are easy, using the built-in capabilities of a ray-tracing program, whose job is mainly to throw and follow rays of light. We use PovRay, a free and open source ray-tracer. Then, outputting the information defining every elementary truncated cone (position, orientation, dimension), we produce a Python script that we feed into Blender, a free and open source 3D modeler. We then scale and manipulate the three-dimensional sculpture, before outputting yet another format that we send to a 3D printer (this is the only non-free part of the process, if we ignore the cost of computations, but that is the price we pay when going from virtual to real!).

5 A Complete Example

Let us illustrate the process, from the original design to the created sculpture. The design we chose is featured in figure 8. Note that the center of this image is free from any line segment: as a practical rule, one must avoid line segments (or circular arcs) passing through or near the center of the image: this results in parts of the sculpture located between the observer and the mirror, thus interfering with the reflection. The image was first scaled to occupy most of the visible surface of the mirror. We then split the image into line segments, which correspond to continuous but distorted curves in the final sculpture. For aesthetic reasons, we ensured that segment endpoints were always on either the first or the last surface of distortion (S_0 or S_{N-1}), and that for each line segment, both of its endpoints lie on opposite surfaces of distortion (one near the mirror: S_0 , the other far from it: S_{N-1}). This was done for the chosen design, as shown in figure 9. This will not always be the case: just consider an image containing a pentagon or any other odd-sided polygon¹.

The mirror we chose was a simple spherical one, but the mirror's actual shape does not influence the process in any way, as long as it is convex (but under certain particular conditions, concave mirrors can also be used). Next, we chose the set of surfaces of distortion: here we simply used a set of fifty concentric spheres, with the same center as the mirror. Again, any other choice would have been valid, provided each P_i has an image on S_i . Having set the observer's position, we called PovRay and produced the virtual anamorphosis setting of figure 10. Figure 11 shows the same installation from another point of view. After having sent the description of the sculpture to a 3D printer, we obtained the object in figure 12. Finally, we validated the whole process by creating the final installation featured on figure 13. The differences between

¹One could object that figure 8 contains triangles, but those are in fact degenerate quadrilaterals, with three points aligned on the hypotenuse.

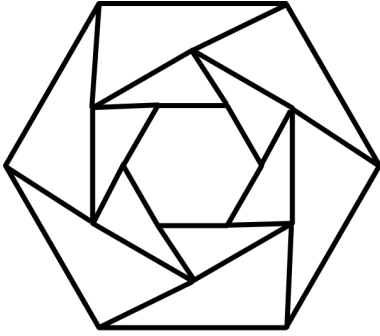


Figure 8 : *The original image.*

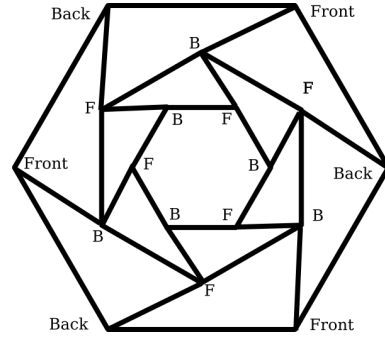


Figure 9 : *Each segment has a Back and a Forward endpoint.*

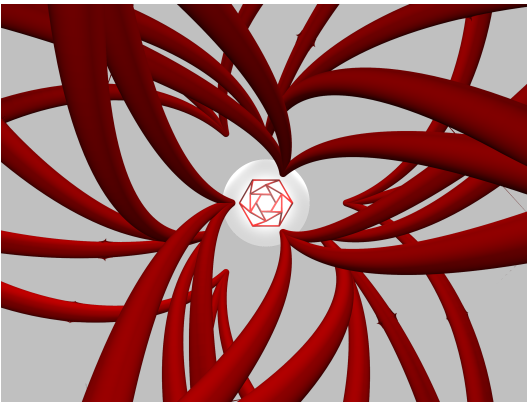


Figure 10 : *The virtual installation.*

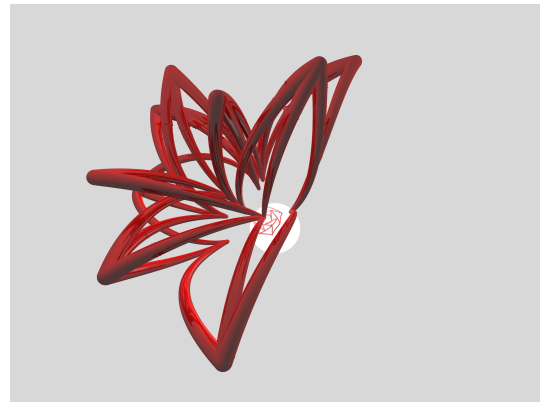


Figure 11 : *A side view of the sculpture.*

figure 10 and figure 13 are only due to the different focal length of the cameras.

6 Conclusion and Perspectives

A New Field of Investigation. Following the validation of this method by the physical creation of an installation, a wide range of experiments could be conducted. The mirror's shape, the position of the observer and the definition of the set of surfaces of distortion define a kind of cartesian product of possible anamorphosis settings. Each of them can be defined and tested at no cost in less than an hour. It takes no more time to translate the virtual design into a suitable file, and then send it to a 3D printer. Using online 3D printing websites, one then has to wait one week to see the physical result, with no surprise: everything is like it is in the virtual version.

Extending the Method Even More. Several variations immediately come to mind: can we replace the wired structure with a solid one, that is to say using surfaces instead of wired skeletons? What if we use a different (random?) set of surfaces of distortion for each line segment of the desired image?

Colombus' Egg. The process described here uses only one idea, namely following rays of light in the same spirit as the method described by Denis Diderot three hundred years ago in his Encyclopædia[4], cited in [3]. All the rest is only a matter of using existing free computer tools. Like a craftsman able to make new objects because he knows how to manipulate his tools, computer scientists are now able to answer questions quickly, with the help of existing tools. The plethora of such programming tools increasingly opens the way

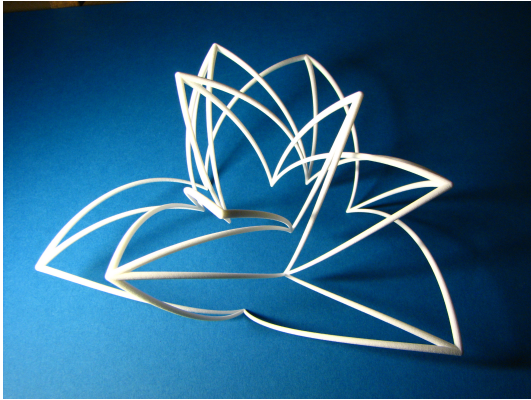


Figure 12 : *The 3D printed sculpture.*



Figure 13 : *The final anamorphosis setting.*

to experimental mathematics. Questions like “What would this mathematical object look like?”, “What if I tune this parameter?” can now have a quick answer, in the form of a drawing, or even of a three-dimensional object.

A Fruitful Collaboration Between Artist and Scientist. Without the original demand from James Hopkins, I think I would never have thought of this modification of the original method described in [3]. Despite its basic definition, the example presented in section 5 shows the unexpected artistic appeal of the distorted objects. True artists, used to manipulating physical shapes and materials, might surely extract much more beauty from this method. I would be glad to collaborate with them, as I am curious to see what kind of shapes and anamorphosis installations they could imagine.

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